Monopoly

Question:
Which properties are the best?

This is open-ended. We won't end up answering this question, but it's a good starting pt.

There are many different ways you could go with this: ~~but there is~~ we will eventually focus ~~on~~ one, but first let's brainstorm.

(. . .)

List factors:

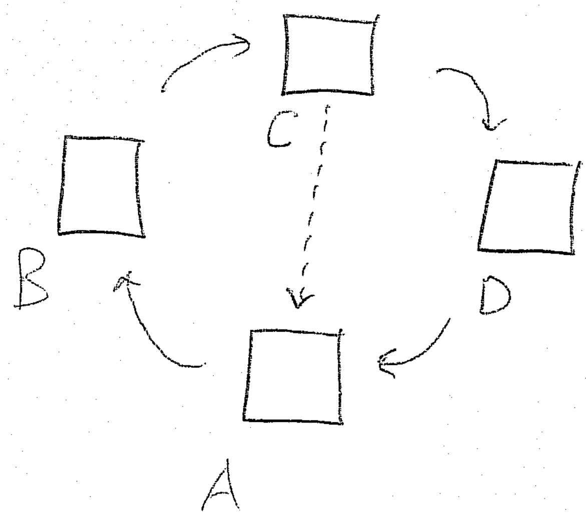
(. — .)

If we accounted for all of these, then the answer would depend on the situation, and we'd really be trying to answer "what should you do at any given (Too

Value of a property:
Balance how much it costs, how much
$ you get when people land on it, and
how much people land on it.
Let's focus on 3rd factor... so,
this is a talk about applying some specific
ideas from probability.
So: Which squares are most likely to
be landed on?
(. . .)

Complication we want to ignore: Everyone
starts in one spot.
What we really want to answer: If a
player has been moving for a while, then
what is the prob. we'll find him in each
particular square?

Let's try this for a much simpler gameboard:



Your "die": $\frac{1}{2}$ prob. moving 1
$\frac{1}{2}$ prob. moving 2

and if you land on C, you automatically go to A.

A --- $\frac{1}{2}$ B
$\frac{1}{2}$ C

B --- $\frac{1}{2}$ C
$\frac{1}{2}$ D

C --- 1 A

D --- $\frac{1}{2}$ A
$\frac{1}{2}$ B

2 ways
compute
solve exactly

Compute

Say we start at A, and see what happens:

$$1\,A \longrightarrow \begin{array}{l} \frac{1}{2} B \\ \frac{1}{2} C \end{array} \longrightarrow \begin{array}{l} \frac{1}{2}\left(\frac{1}{2} C + \frac{1}{2} D\right) \\ \frac{1}{2}(1A) \end{array} = \begin{array}{l} \frac{1}{2} A \\ \frac{1}{4} C \\ \frac{1}{4} D \end{array} \longrightarrow \begin{array}{l} \frac{1}{2}\left(\frac{1}{2} B + \frac{1}{2} C\right) \\ + \frac{1}{4}(1A) \\ + \frac{1}{4}\left(\frac{1}{2} A + \frac{1}{2} B\right) \end{array} = \begin{array}{l} \frac{3}{8} A \\ \frac{3}{8} B \\ \frac{1}{4} C \end{array}$$

$$\longrightarrow \begin{array}{l} \frac{3}{8}\left(\frac{1}{2} B + \frac{1}{2} C\right) \\ + \frac{3}{8}\left(\frac{1}{2} C + \frac{1}{2} D\right) \\ + \frac{1}{4}(1A) \end{array} = \begin{array}{l} \frac{1}{4} A \\ \frac{3}{16} B \\ \frac{3}{8} C \\ \frac{3}{16} D \end{array} \longrightarrow \begin{array}{l} \frac{1}{4}\left(\frac{1}{2} B + \frac{1}{2} C\right) \\ \frac{3}{16}\left(\frac{1}{2} C + \frac{1}{2} D\right) \\ \frac{3}{8}(1A) \\ \frac{3}{16}\left(\frac{1}{2} A + \frac{1}{2} B\right) \end{array} = \begin{array}{l} \frac{15}{32} A \\ \frac{7}{32} B \\ \frac{7}{32} C \\ \frac{3}{32} D \end{array} \longrightarrow \cdots$$

tedious.

$$\xrightarrow{\times 50} \begin{array}{l} .3529 \\ .2353 \\ .2941 \\ .1176 \end{array}$$

Is there another way?

· Solve exactly

Well, if we found the right probabilities, then if we start with that mix of probabilities and move the system forward one step, the mix should stay the same.

[Justify this.]

So, we want p_A, p_B, p_C, p_D s.t.

$$\begin{array}{l} p_A A \\ p_B B \\ p_C C \\ p_D D \end{array} \longrightarrow \begin{array}{l} p_A\left(\tfrac{1}{2}B+\tfrac{1}{2}C\right) \\ p_B\left(\tfrac{1}{2}C+\tfrac{1}{2}D\right) \\ p_C\,(A) \\ p_D\left(\tfrac{1}{2}A+\tfrac{1}{2}B\right) \end{array} = \begin{array}{l} \left(p_C+\tfrac{1}{2}p_D\right)A \\ \left(\tfrac{1}{2}p_A+\tfrac{1}{2}p_D\right)B \\ \left(\tfrac{1}{2}p_A+\tfrac{1}{2}p_B\right)C \\ \left(\tfrac{1}{2}p_B\right)D \end{array} = \begin{array}{l} p_A\ A \\ p_B\ B \\ p_C\ C \\ p_D\ D \end{array}$$

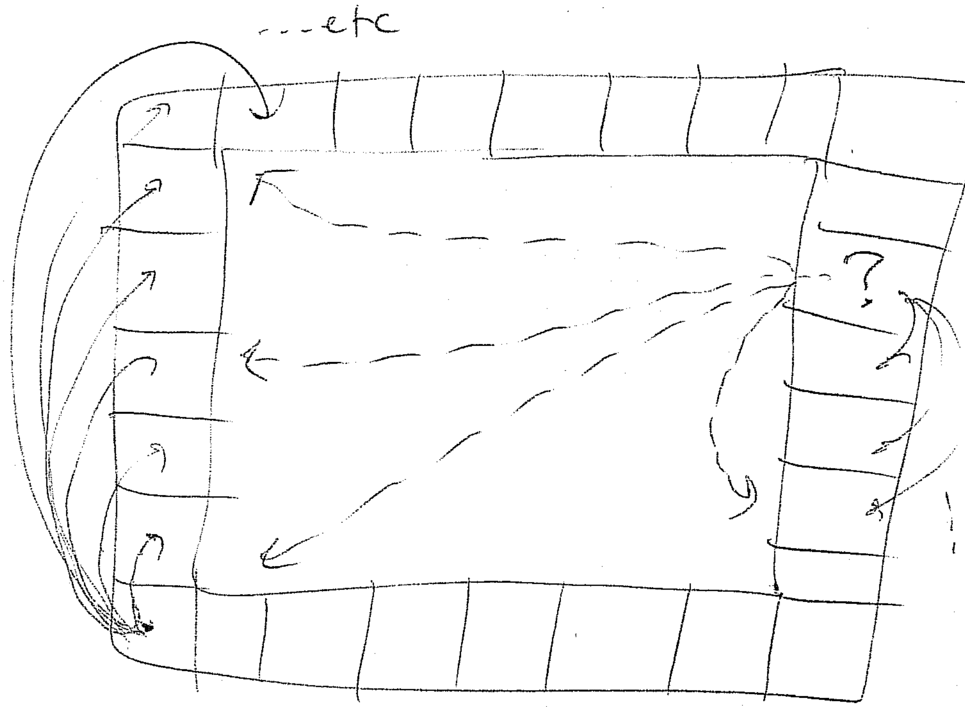
$$\begin{aligned} p_A &= p_C + \tfrac{1}{2}p_D \\ p_B &= \tfrac{1}{2}p_A + \tfrac{1}{2}p_D \\ p_C &= \tfrac{1}{2}p_A + \tfrac{1}{2}p_B \\ p_D &= \tfrac{1}{2}p_B \end{aligned}$$

$\rightsquigarrow$ Could solve, but if system gets more complicated, this isn't doable.

$$\boxed{p_A = \frac{6}{17},\quad p_B = \frac{4}{17},\quad p_C = \frac{5}{17},\quad p_D = \frac{2}{17}}$$

That's too hard.
Let's use the first idea, namely start somewhere and keep running the system forward in time, but let a computer do the heavy lifting.

Our situation looks more like



This, when you have a finite number of possible positions, and "transition probabilities" telling you the prob. of going from one position to another, is called a <u>Markov Chain</u>. Can be solved in the same <u>compute</u>.

Illinois	.0272
B&O	.0262
Tenesse	.0256
Water Works	.0254
New York	.0251

Mediterr.	.0189
Park Place	.0191
Baltic	.0193
States	.0198
Oriental	.0201

col =

0.0271	1
0.0189	2
0.0192	3
0.0193	4
0.0207	5
0.0238	6
0.0201	7
0.0204	8
0.0204	9
0.0202	10
0.0378	11
0.0237	12
0.0231	13
0.0198	14
0.0223	15
0.0237	16
0.0243	17
0.0232	18
0.0256	19
0.0251	20
0.0258	21
0.0235	22
0.0256	23
0.0233	24
0.0272	25
0.0262	26
0.0230	27
0.0228	28
0.0254	29
0.0222	30
0.0227	31
0.0230	32
0.0226	33
0.0233	34
0.0216	35
0.0236	36
0.0201	37
0.0191	38
0.0191	39
0.0231	40
0.0309	41
0.0257	42
0.0215	43