Mathematical Card Tricks

Much of this is taken directly from [3][pp57-58] or the Internet. There are also a collections of books by Karl Fulves that illustrate a lot of these such tricks, see [1].

1 Some Tricks

1. **Compatibility test.** Found in [1]. This trick has a "story" that helps focus the attention away from the cards. Here is the basic idea, you figure out how it works.

   (a) Two volunteers are picked from the audience. The story works if the two volunteers are married or boyfriend-girlfriend.

   (b) Volunteer 1 shuffles the deck, deal any 8 cards, then 7 more, then 6 more then 5. These cards are all then put into a single pile. The rest of the cards are given to Volunteer 2.

   (c) Volunteer 1 turns her cards face up but Volunteer 2 had his cards face down.

   (d) Volunteer 1 deals her cards into two piles: pile A with the red cards and pile B with the black cards.

   (e) While Volunteer 1 is dealing, Volunteer 2 "matches" the deals make by Volunteer 1. In other words, Volunteer 2 also deals two piles: piles C and D (but does not look at the faces). If Volunteer 1 deals a card in pile A then Volunteer 2 deals a card into pile C. Similarly, for each card Volunteer 1 deals into pile B, Volunteer 2 deals a card into pile D.

   (f) When finished, there are now two face-up piles and and two matching face-down piles.

   (g) The magician now makes a silly speech about the compatibility between the two volunteers:

   > "These face-up cards represent qualities we can all see in someone. For example one person is tall the other short. One is quiet, the other outgoing."
   > "These face-down cards represent hidden qualities—personality, character, spirit. We are going to focus on the red cards here (points to pile C) and the black cards here (points to pile D). If there are far more reds in the one pile than blacks in the other then it means the two are incompatible. The closer the numbers

are together are together, the better the two people match up. On very rare occasions, perhaps one in a million, the two people are so well matched up that there are exactly the same number of reds and blacks."

(h) At this point, the cards are counted and the two volunteers are shown to be compatible.

How does it work?

2. **Psi-X.** Found in [1].

   The magician takes a deck of cards and a volunteer from the audience.

   (a) The volunteer cuts part of the deck and shuffles that portion of the deck and places the cards on the table.
   (b) The volunteer takes the first card on top of the unshuffled portion of the deck, looks at it and places it on the top of the shuffled portion of the deck.
   (c) The volunteer then shuffles the unshuffled portion of the deck and places it on top of the other deck. The selected card is trapped between the two halves of the deck.
   (d) The magician then looks at the deck and selects the card (adding in a bit of magical flair to make it look even better).

   How is this done?

3. **Shuffle pick-the-card trick.** This is called *Any Deck, Any Time* and can be found in [2]. Here is how it works.

   (a) A volunteer takes a deck of cards and divides them into three approximately equal face down piles on the table.
   (b) The volunteer takes one of the piles, shuffles it and notes the top card. (The shuffle is irrelevant but helps the audience feel things are mixed up.)
   (c) The volunteer takes this pile and puts it face up on one of the other piles and the places the third pile face down on top. Thus, there is a portion of face-up cards sandwiched between face-down cards.
   (d) The volunteer now takes the cards, cuts them and shuffles them together.
   (e) The magician now takes the cards and finds the selected card.

   How is this done?

4. **Magic card and magic number**

   Shuffle a deck of cards and note the bottom card. Call this card the "magic card." Deal 12 cards face down and have someone turn over any four cards. Place the remaining eight cards on the bottom of the deck Assigning values 1 to and Ace, 10 to a Jack, Queen or King, and the face value to a number card, sum the values on the four selected cards. Call this the "magic number."

   On top of each selected card deal, in turn, the number of cards required to increase the face value of the card to 10. For example, on top of a 2 you would place 8 cards. On top of a 7 three cards, and on top of a Jack zero cards. Collect all four piles of cards and place them on the bottom of the deck. Now deal out the "magic number" of cards from the top of the deck. What is the final card dealt.

5. **Seven rows of three**

   Lay out 21 cards face up in a grid of seven rows and three columns. Have a friend mentally select one card and indicate to you the column in which the card lies. Pick up the cards one column at a time, carefully preserving the order within the columns, and making sure to collect the indicated column second.

   Lay out the 21 cards again, row by row, to obtain seven rows of three. Have your friend indicate to you the column in which the selected card lies. Pick up the cards as above and repeat the process one more time.

   Pick up the cards again as above. Deal ten cards from the top of the pile and hand the 11th card to your friend.

6. **Guessing the card**

   Ask your friend to shuffle a pack of cards and then select any spot card (not a picture card). Turn your back so that you can't see what he is doing then ask him to place the selected card face down on top of the squared up pack. Now tell him to remove its value in cards from the bottom of the pack and place them on top of it - for instance if he chose the Seven of clubs, he simply takes seven cards from the bottom and puts them on the top, above his card. He must do this silently giving no clue as to the number of cards he is moving.

   Next ask him to deal the cards from the top of the pack one at a time on to the table, and call out the name of each card in turn. He must deal the cards without giving any clue in his voice until he has gone right past his own card until you tell him to stop. Then you name his card. How do you do it?

7. **Deal and double cross**

The performer asks a spectator to tell him to mix up some cards, following the performer's instructions to the letter. No matter how much they're mixed up, they end up in the same order as at the beginning of the trick.

Before you begin the trick. Take a full suit of 13 (any one) and lay them down on the table, face up, in numerical order. Tell a spectator to note the order of the cards. Square them up and hold them face down in your left hand.

Here's how it's done. Having first shown the spectator the suit of cards in numerical order and then taken them back into your hand, keeping them face down.

You say, "You must tell me exactly how to deal these cards so we can be sure they're thoroughly mixed."

Explain that the instructions you need from them are "DEAL" and "DOUBLE CROSS." When they say "DEAL" you'll put one card on the table (face down) and when they : "DOUBLE CROSS" you'll take the next card from the pile in your hand, put it under the first one (on the same pile) and then deal both of them together on the table (again face down).

You then follow their instructions and deal the cards exactly as they say into a pile on the table.

You square up the pile and then suggest that to be sure the cards are really well mixed, you do it once more.

Once again you follow their instructions and deal the cards as they say into a pile.

Then, picking up the pile you say, "You're now going to see that no matter what you tell me to do with the cards, they will only do as I tell them!"

Turn the cards over so they're face up and spread them out in a line. All 13 of them will be in the same numerical order as at the beginning of the trick!

2 Activities

Your job is to explain what is going on in each of these cases.

1. **Two piles of 26**

   Shuffle the cards and divide them into two piles of 26 cards each. Count the number of black cards in the first pile and the number of red cards in the second pile. What do you notice?

2. **Piles of 32 and 20**

   Shuffle a deck and divide them into two piles, one of 32 card, the other of 20 cards. Count the number of black cards in the first pile and the number of red cards in the second pile. Subtract the smaller number from the larger number. What do you get?

3. **Two piles of 26, part 2**

   Split a deck of cards into two piles according to color. Take ten cards from the red pile and shuffle them into the black pile. Without peeking, select ten cards from the (mostly) black pile and place them into the red pile. Count the number of "foreign" cards in each pile. What do you notice?

4. **Cutting and flipping the deck**

   Shuffle a deck of cards, take note of the top card, and place the deck face down on the table. Cut the deck, flip the top pile over and place it back on the deck. Cut the deck again, deeper this time, and again flip the top pile over and place it back on the deck. Now remove all the cards face up. What is the next card?

5. **Ordered deck and then shuffled by cutting**

   Arrange a deck of cards so that all cards of the same suit appear in order from Ace, King, Queen, down to two. Each suit contains 13 cards. Cut the deck 13 times. Deal out, face down, a row of 13 cards from left to right. On top of them deal another row of 13 cards from left to right. Repeat this two more times until all the cards are dealt. Flip over each pile of four, what do you notice?

6. **Hall's Marriage Problem**

   Shuffle a deck of cards and divide them into two equal piles of 26 cards. Clearly it is always possible to select a red card from one pile and a black card from the other. But suppose these cards were dealt instead into four piles of 13 cards. Would it be possible to select a spade from one pile, a club from another, a heart from a third and a diamond from a fourth? (Try it!) Is it always possible to accomplish this feat no matter how the cards are distributed?

   How about this variation: Deal out thirteen piles of four cards each. Is it always possible to select an ace from one pile, a two from another, all the way to a king from a 13th pile?

   Here is another variation: Deal out 26 piles of two cards each. Is it possible to select a black ace from one pile, a red ace from another, a black two from a third, a red two from a fourth, and so on to a black king and a red king from the remaining two piles?

7. **Perfect Shuffling**

A perfect shuffle is when the deck is divided in exactly two halves and the cards are alternately interlaced into a single pile. We will only worry about 8 cards, numbered 1 through 8. There are two ways to alternate these cards as in Figure 1.

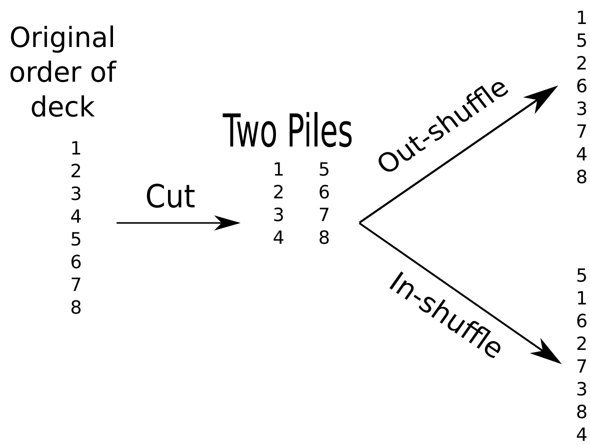

Figure 1: Perfect Shuffles

(a) Using a deck of 8 cards, what happens if you perform a perfect out-shuffle three times?

(b) Again, using a deck of 8 cards. Using a combination of in- and out-shuffles, is it possible to move the top card to an arbitrary position in the deck?

References

[1] Fulves, Karl, *My best self-working card tricks*, Dover Publications, Mineola, N.Y., 2001.

[2] Fulves, Karl, *Self-working card tricks*, Dover Publications, Mineola, N.Y., 1976.

[3] Tanton, James, *Solve this, math activities for students and clubs*, Mathematical Association of America, Washington, D.C., 2001.