

# Olympiad Middle School

WU Math Circle

April 25, 2010

1. Express  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$  as a common fraction.
2. A cube has edge length 11, what is its volume?
3. A right triangle has sides all with integer length. The first side is length 5 and the second is length 3, what is the length of the third side?
4. When divided by 23, the number 43255 has remainder 15 and the number 55234 has remainder 11. What is the remainder when  $43255 + 55234 = 98489$  is divided by 23?
5. A certain shirt regularly costs \$200. Jeff buys it during a 15% off sale and uses a coupon to get an additional 20% off of the sale price. What does Jeff pay for the shirt?
6. If the numbers 1, 2, 3, and 12 are used only once in the expression  $\frac{a}{b} + \frac{c}{d}$ , what is the smallest sum that can be obtained?
7. Jenny has found the perfect purple paint by mixing 3 tbsp of blue paint and 6 tbsp of red paint. If Jenny wants to make 12 gallons of purple paint, how many gallons of red paint must she use?
8. After being painted, a solid wooden cube whose edge is 4 cm is cut into 64 small 1-cm cubes. How many of these small cubes have exactly two painted faces?
9. What is the largest possible area of a rectangle with integer sides and a perimeter of 30?

10. The points  $A, B, C,$  and  $D$  all lie on the same line in that order. The distance from  $A$  to  $B$  is 3, the distance from  $B$  to  $D$  is 15, and the distance from  $A$  to  $C$  is 8. What is the ratio of the distance between  $B$  and  $C$  to the distance between  $A$  and  $D$ ?
11. What is the next number in the sequence  $1, 5, 13, 29, 61, \dots$ ?
12. A full bottle of milk weighs 40 oz. and a half full bottle of milk weighs 24 oz. How much does the empty bottle weigh?
13. What is the remainder when  $2010^{234}$  is divided by 9?
14. Three siblings were born three years after each other. The eldest is exactly two times as old as the youngest. How old is the middle sibling?
15. How many cards must you draw from a deck of 52 cards to be certain that you have 6 hearts?
16. How many two digit numbers are divisible by both 3 and 5?
17. How many unique lists of length three can you make with the numbers 1, 2, 3? ( $2, 1, 2$  and  $2, 2, 1$  are different lists)
18. What is  $405 \times 395$ ?
19. On a number line,  $M$  is halfway between  $A$  and  $B$ . If  $A = -3$  and  $M = 5$ , then what is  $B$ ?
20. Five students (Dena, Lucy, Carl, Lou, and Heather) will stand in a line. If Carl and Lou must stand next to each other, and Lucy must be first, in how many possible spots can Heather stand?
21. Dwayne participated in a free-throw shooting tournament. Each contestant was given 10 chances to make baskets from the free-throw line. If Dwayne makes a basket, he receives 3 points. If he misses the basket, he loses 1 points. Dwaynes final score was 14 points. How many baskets did he make?
22. What is the largest three digit number such that the product of the digits is 9?

23. The time in St. Louis is 2 hours later than the time in Seattle. If a plane leaves St. Louis at 3 pm and it takes 4 hr 35 min to fly from St. Louis to Seattle, what is the local time when the plane lands?
24. Which is smaller  $\frac{14}{20}$  or  $\frac{36}{50}$ ?
25. What is the area of a right triangle which has integer sides such that one side of length 6 and another of length 10?
26. What is the largest two digit prime number?
27. Larry lives 2 miles from the store. He can walk to the store in 36 minutes. However, he walks home from the store at  $\frac{2}{3}$  that speed due to the weight of the groceries. If Larry spends 40 minutes in the store, how long is his trip?
28. What is the greatest common factor of 432 and 234?
29. We have a thick ring (annulus) whose inner radius is 2cm and outer radius is 3cm. What is its area?
30. What is  $\sqrt{(\sqrt{3})^2 + (\sqrt{6})^2}$ ?