Math Circle

Voting Methods Practice

March 31, 2013

1) Three students are running for class vice president: Chad, Courtney and Gwyn. Each student ranked the candidates in order of preference. The chart below shows the results of the voting.

	6 boys	4 boys	8 girls	4 girls
First choice	Chad	Chad	Gwyn	Courtney
Second choice	Courtney	Gwyn	Courtney	Gwyn
Third choice	Gwyn	Courtney	Chad	Chad

Notes on reading the chart: The first row tells you how many students voted with a ranking that is indicated by the corresponding column. For example, the entry in the first row, last column tells you that four girls turned in ballots with 1-Courtney, 2-Gwyn, and 3-Chad.

(a) How many students voted? How many boys? How many girls?

(b) Use the Plurality Voting Method to determine who wins the election.

(c) Use the Vote-for-Two Method to determine the winner.

(d) Use the Borda Count Method to determine the class's new vice president.

(e) Does anything seem strange?

(f) If you had voted 1-Chad, 2-Courtney, 3-Gwyn, which voting method would you prefer? Why?

(g) Does the fact that there are more boys than girls in the class seem to affect the outcome for any of the methods?

Math Circle

Transitivity and Condorcet Winners

March 31, 2013

If a person prefers A over B, and B over C, then it stands to reason that she must prefer A over C. But when more than one person expresses their preferences for three or more alternatives, odd things can happen that might not seem reasonable.

Transitivity: If $a > b$ and $b > c$, then $a > c$. This works for real numbers, and for an individual's preferences in an election with three candidates.

For example, let's say there is going to be a big celebration dinner for the Math Circle participants and their families. There will be sixty people attending. Professor Thornton books the party room at a fancy restaurant. In order to keep things easier to manage, there will be only one menu offering. The choices are salmon or chicken. Professor Thornton asks everyone to submit their preferences, and will use these to select the dinner that the majority prefers.

The results are that 40 people voted for salmon, and 20 for chicken.

So Blake intends to order salmon. But when he calls the restaurant, the owner says that there has been a problem with their fish guy, and now the choice is chicken or beef.

It turns out that 45 people would prefer chicken, and 15 would prefer beef.

So Blake orders the chicken, and considers the job done. But the next day, the owner calls to say that she has a new fish guy that she really likes, and there is a salmonella scare, so many people are avoiding chicken. She wants to know if Blake would rather switch back to the group's original first choice, salmon. He is tempted to just say yes, but knows that since the students have just learned a bunch about voting, they will be happy to vote again, so he asks them which they prefer: salmon or chicken?

To his surprise, these are the results: 35 people prefer beef, and 25 prefer salmon.

How can this be?

1) Can you complete this chart to accurately illustrate the voting preferences of the voters?

	25 people	people	15 people
First choice	salmon		beef
Second choice		beef	salmon
Third choice			chicken

2) What if the 15 people in the last column had actually voted for 1) beef, 2) chicken, 3) salmon?

	25 people	people	15 people
First choice	salmon		beef
Second choice		beef	chicken
Third choice			salmon

In this case, chicken would be called the Condorcet winner. That is, if chicken were pitted in a one-on-one race against any one of the other choices, it would win. Unfortunately, as we just saw, there isn't always a Condorcet winner.

3) Look at the results from the classroom voting on the first worksheet. How many students preferred Courtney over Chad? How many students preferred Chad over Gwyn? How many students preferred Courtney over Gwyn?

4) Is Courtney a Condorcet winner? Is there a Condorcet winner in that election?

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Arrow's Impossibility Theorem

March 31, 2013

Arrow's Impossibility Theorem (Starbird/Berger)

When there are three or more alternatives, it is impossible to devise any voting scheme (that is, a rule for taking the voters' transitively ranked orderings of the candidates and producing one cumulative ranking for the group) other than a dictatorship that will order the alternatives or select the winner in a manner that satisfies the following principles:

1) Go along with the consensus–If it is unanimous, then we have a winner.

2) Throwing out a loser will not change the election.

3) Changing a vote to favor a candidate will not result in that candidate's ranking dropping.

Example 1: It is possible in the runoff scenario 1 that Sarah might be preferred by all of the students over the eventual winner. This illustrates how principle 1 might fail.

Example 2: Many people believe that, had Ralph Nader dropped out of the 2000 U.S. presidential race, Al Gore would have won the election. This illustrates how principle 2 might fail.

Example 3: Classroom runoff example shows how principle 3 might fail.

Applications: Economics, Social Choices, Game Theory

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Runoffs and Throwing Out Losers Examples

March 31, 2013

The class from the first example is going to have a new election. This time, there are two boys and three girls running for the position: Chad, Cliff, Courtney, Gwyn and Sarah. Recall that there are 10 boys in the class, and 12 girls.

The new election will be held as a runoff. This means that there will be two rounds of voting. For the first round, the voters express their first choice, and the two candidates with the most votes go on to the second round. The candidate with the most votes after the second round wins.

1) Suppose the boys' votes are split evenly between Chad and Cliff, and the girls' votes are split evenly among the three female candidates. Who will be in the runoff?

Let's say that the second round is very close, but Chad manages to win over Cliff.

2) Suppose that before the first round of voting, Chad campaigns harder, and gets more than 5 of the boys' votes. Who will be in the runoff this time?

The girl that is running against Chad has the backing of all the other girls in the class, and handily beats him in the runoff.

3) Does this seem fair? Originally, she might have had fewer votes than Chad when there were five candidates.

Recall: One of Arrow's principles is that if a voter changes his or her vote to now favor a given candidate (as at least one of the boys did), then that change should not lead to a worse result for that candidate. So the runoff method doesn't satisfy that principle.

Now the class is going to try another voting method for the election involving the five candidates Chad, Cliff, Courtney, Gwyn and Sarah. We will call this method Throwing Out Losers. Each voter indicates their **least** favorite candidate. The candidate who gets the most of the least favorite votes is eliminated, then the process is repeated until there is a winner.

4) Suppose that Sarah is the first choice of each girl, but the boys like her the least. The girls equally dislike Chad and Cliff more than the other candidates. Is there a way for Chad to win this election? Explain.

5) Even though Sarah was the first choice of most of the class, there is no way for her to win this election. Does that seem fair?

6) Suppose that before the election is held, Cliff decides not to run. Assuming that they follow the pattern already established, how will the girls vote in the first round?

7) What happened to Chad? He would have won if Cliff (who didn't have a chance of winning) had stayed in the election. Does that seem fair?

This shows how the second principle of Arrow's Theorem can fail. Throwing out a loser changed the result. It also shows that this method doesn't necessarily reflect the preference of the majority of the group, since Sarah could lose even though she would beat any other candidate in a head-to-head contest.