MATH CIRCLE

Genival Jr.

(The following text is an outline for the October 20th Math Circle in WUSTL)

The Following problems are known as THE HAT COLOR PROBLEM. Basically there are n players and a hat will be put in the head of each player, also each hat is one of k given colors. Players can see others hat colors but not their own, the winning condition is given in each problem. In order to simulate the situations, you will need addtional players to form a team, please invite someone to join your team.

1. First we have 2 players and 2 hat colors say red and blue. Each player is assigned a hat either red or blue. They will simultaneously guess their hat color or pass, if at least one of the players guess his hat color right they win. Find a strategy to always win.

2. Celcius, Quintus, and Meridius sit in a circle, blindfolded. Hats either red or blue are placed on their heads. When the blindfolds are removed they are asked to raise a hand if they see a red hat on one or both of his friends. Based solely on the information provided by these gestures is it possible for at least one man to correctly announce the color of his own hat (if players agree to keep silent unless they are certain of their answer)?

3. Same situation as in 2. but with 4 players and again two colors.

4.This time 50 blindfolded gnomes stand in a line, back to front. Each gnome's hat is either red or blue and therre is at least one red hat among the gnomes (and the gnomes know this). Upon the removal of the blindfolds each gnome can see the hats of the gnomes in front of him, but not behind him. (Thus the gnome at the back of the line can see 49 hats, the gnome at the front, none.) Starting at the back of the line, each gnome, in turn, will be asked to say either My hat is red or pass. The game will endpositivelyas soon as some gnome makes the former statement and is correct! If a gnome incorrectly claims his hat is red, all will be eliminated. If all gnomes choose to say pass, then all shall be eliminated. Devise a strategy that guarantees the gnomes survival.

5. Now we have 4 players and 4 hats, 3 hats are red and 1 hat is blue. If the players are A, B ,C,D and only A,B,C can see each other but they can't see D and D can't see them. If at least one of the players guess his hat color they win. Find a winning strategy.

6. Same as in 5. but this time there are 2 red hats and 2 blue hats and B can only see C, C can see nobody's hat, A can see B and C and D again can

see nobody's hat. Find a winning strategy.

7. In another variant, only three players and five hats (supposedly two black and three white) are involved. The three players are ordered to stand in a straight line facing the front, with A in front and C at the back. They are told that there will be two black hats and three white hats. One hat is then put on each player's head; each player can only see the hats of the people in front of him and not on his own. The first player that is able to announce the colour of his hat correctly will be released. No communication between the players is allowed. After some time, only A is able to announce (correctly) that his hat is white. Why is that so?