Rules of Chomp

- You are given a graph which consists of dots and lines.
- Players take turns either removing a single line or removing a dot and all connected lines.
- The winner is the player to remove the last dot.
EXAMPLE GAME

player 1 takes a dot

player 2 takes a line

player 1 takes a dot

player 2 takes a dot

player 1 wins
GAME 1

GAME 2
Questions

- Is there a strategy for Player 1 to always win Game 1?
- What about Player 2?

- What about Game 2?
Games 1 & 2
Use linear graphs:

...:

Question:
Is there a strategy so that Player 1 can win on any linear graph?
Answer:

Player 1 removes the middle dot (if odd # of dots) or middle line (if even # of dots)

Then, Player 1 mimics Player 2 on the remaining 2 identical graphs.
GAME 3

GAME 4
Games 3 & 4
Use "Star" Graphs

Question: Is there a strategy so that Player 1 always wins with a Star Graph?
ANSWER:

IF ODD # OF DOTS, PLAYER 1 REMOVES CENTER DOT

IF EVEN # OF DOTS, YOU CAN ALWAYS FORCE YOUR OPPONENT TO HAVE EVEN # DOTS & LINES
GAMES

GAME 6
All graphs so far have been "trees" — meaning the graph is connected and contains no loops or cycles.

Question: Can Player 1 always win Chomp on a tree?
STRATEGY FOR PLAYER 1 TO WIN ON A TREE:

ALWAYS FORCE OPPONENT TO HAVE EVEN # OF DOTS AND LINES

CAN THIS ALWAYS BE DONE?

IMPORTANT HINTS:

• ON A TREE, \[ \# \text{ DOTS} = \# \text{ LINES} + 1 \] \( \text{(WHY?)} \)

• ON ANY GRAPH WITH ODD # DOTS, SOME DOT HAS EVEN # OF LINES ATTACHED \( \text{(WHY?)} \)
GAME 7

GAME 8
QUESTION:

DOES PLAYER 1 HAVE A STRATEGY TO ALWAYS WIN GAMES 7 & 8?
WHAT ABOUT PLAYER 2?

GAMES 7 & 8 USE A "CYCLE GRAPH"

WHICH PLAYER CAN ALWAYS WIN CHOMP ON A CYCLE?
GAME 9

GAME 10
Which player has a strategy to always win game 9? Game 10?

Potential strategy:
Always force your opponent to have even # dots and even # lines
Games 9 & 10 use graphs called "forests" — meaning several separate trees.

Which player always has a winning strategy playing Chomp on a forest?

<table>
<thead>
<tr>
<th>Even # Dots</th>
<th>Odd # Dots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even # Lines</td>
<td>?</td>
</tr>
<tr>
<td>Odd # Lines</td>
<td>?</td>
</tr>
</tbody>
</table>
GAME 11:

GAME 12:
Games 11 & 12
Use "bipartite" graphs:

Does the strategy from before work?

Which player has a winning strategy with a bipartite graph?

<table>
<thead>
<tr>
<th>Even # Dots</th>
<th>Odd # Dots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even # Lines</td>
<td>Even # Lines</td>
</tr>
</tbody>
</table>
GAME 13:

GAME 14:
GAMES 13 & 14
USE COMPLETE
GRAPHS:

PLAYER 1 ALWAYS
HAS A WINNING
STRATEGY ...
UNLESS
# DOTS = MULTIPLE
OF 3