

Ex1. Suppose there are four different blue shirts and six different red shirts. How many ways are there to buy one shirt? How many ways are there to buy a blue shirt and a red shirt?

A: There are $4+6=10$ shirts altogether. So there are 10 ways to buy one shirt.

There are 4 blue shirts and 6 red shirts. So we can pick 1 blue shirt out of 4, and 1 red shirt out of 6. Since picking blue shirt is independent of picking red shirt. There are $4*6=24$ ways to buy a blue shirt and a red shirt.

Ex2. Suppose there are 3 buses that goes from city A to city B, 6 buses from city B to city C, and 7 subway lines that goes from city A to city C. How many ways are there to get to city C from city A?

A: There are $3*6$ ways if we take the bus and 7 ways if we take the subway. So altogether there are $3*6+7=25$ ways to go to city C from city A.

1. How many 4-digit passwords are there with each digit being numbers from 0 to 9?
2. How many 4-digit passwords are there with each digit being numbers from 0 to 9 and neighbor digits are different?
3. How many 4-digit passwords are there with each digit being numbers from 0 to 9, 2nd digit not 7 and last digit odd?

1. How many different ways can we make a queue of size 6 from 7 people?
2. How many different ways can we make a circle of size 6 from 7 people? What about making a circle of size m from n people where n is greater than m ? (two circles are considered the same if everyone has the same neighbors)
3. How many different ways can we make 3 boys and 3 girls sit in a circle such that no two boys sit next to each other?

1. How many different ways can we put 9 distinct balls evenly into 3 distinct buckets?
2. How many different ways can we put 9 distinct balls evenly into 3 identical buckets?
3. How many different ways can we put 9 identical balls into 3 distinct buckets? (does not have to be evenly)
4. How many different ways can we put 9 identical balls into 3 identical buckets? (does not have to be evenly)
5. How many different ways can we put 6 distinct balls unevenly into 2 distinct buckets? What about 6 distinct balls into 3 distinct buckets? Identical buckets?

1. How many ways are there to rearrange the word CALCULUS? What about the word EXISTENCE? And HOLOMORPHIC?
2. How many nonnegative integral solutions are there of the equation $x+y+z+w=30$? And how many of those satisfies $x>1$ and $w>7$?
3. How many ways can you buy three bottles of beverages in a store selling five kinds of beverages? What about five bottles of beverages in a store selling three kinds of beverages?
4. How many ways are there to form 4 teams, each of two people, from a group of eight people? What about in general to form n teams, each of m people, from a group of $m*n$ people?
5. Let n be an integer. Take a sequence of n integers between 1 and n at random. What is the possibility that the sequence contains exactly $n-2$ integers? What is the possibility that the sequence contains at most $n-3$ integers?
6. Given $2n+1$ identical books to put into a bookcase with three shelves. How many ways can we do this such that the books in any two shelves are no more than the books in the other shelf?

Fruit for thought: Given a 3×3 diagram. How many ways are there to fill in numbers 1-9 such that each number appears once, also, numbers are arranged in increasing order when counting from left to right and from up to down.