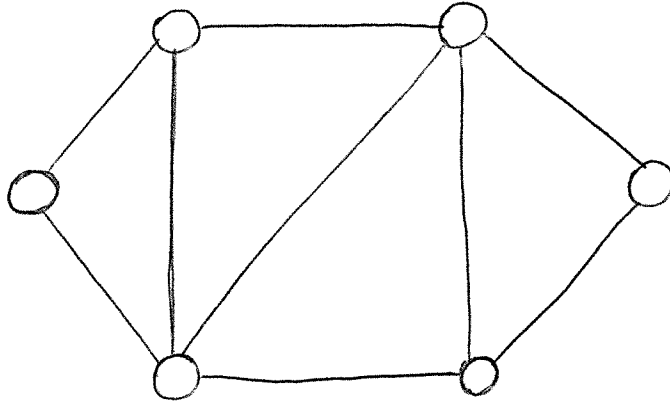
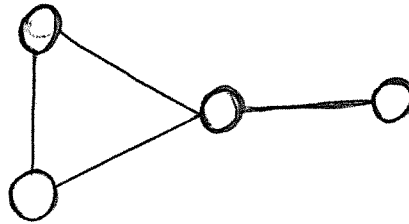


2. MORE COLORING

Question 2.1. Color this graph. Try to use as few colors as possible.



Question 2.2. How many different ways can you color the graph below using 3 colors?



Question 2.3. What is the least amount of colors you can use?

3. COMPLETE GRAPHS

Definition 3.1. A complete graph has an edge between every 2 vertices. It is called complete, because you can not add any more edges. The complete graph with n vertices is called K_n . For example, K_3 and K_4 are:

Question 3.2. Draw K_6 .

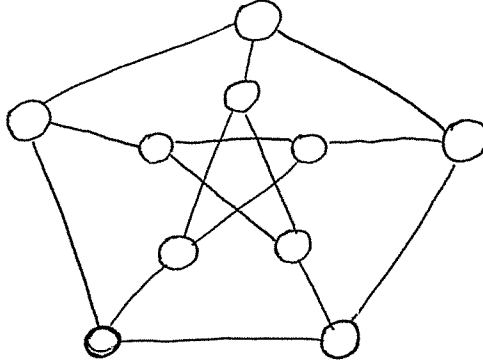
Question 3.3. What is the least number of colors to color K_n ?

Question 3.4. How many edges does K_n have?

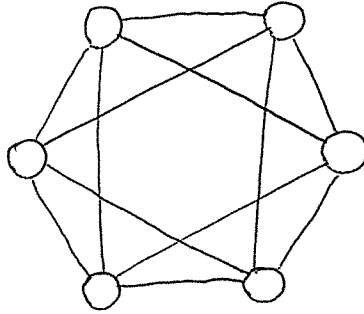
4. CHROMATIC NUMBER

Definition 4.1. The least number of colors to color a graph G is called *chromatic number*, and we denote it by $\chi(G)$

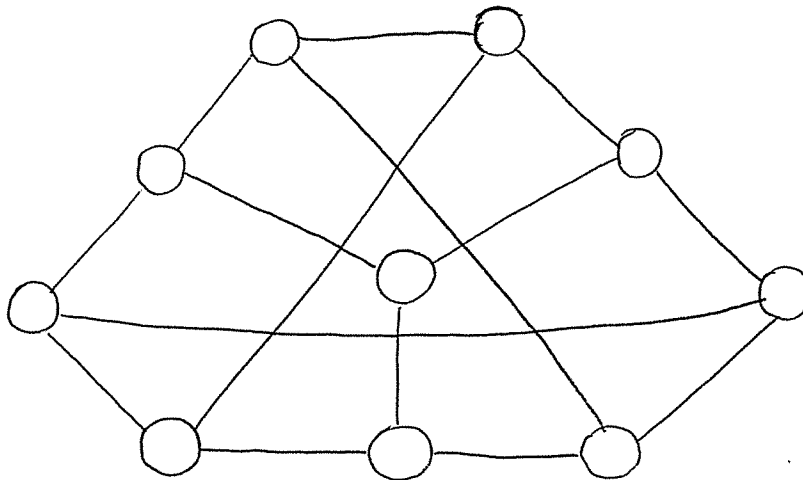
Question 4.2. What is chromatic number of the graph below?



Question 4.3. What is chromatic number of the graph below?



Question 4.4. What is chromatic number of the graph below?

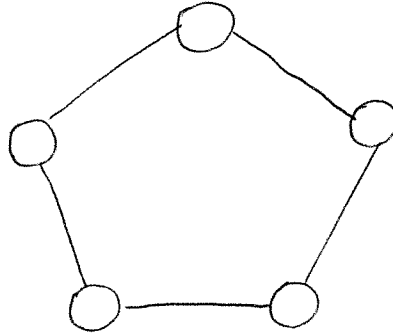


5. YOUR TURN!

Draw a graph below (with at least 10 vertices) and ask someone to color it using the least number of colors.

6. CIRCLE GRAPH

Definition 6.1. A circle graph C_n is the graph with n vertices and edges only on the outside. For example, this is C_5 :

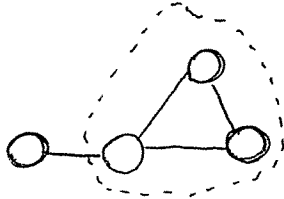


Question 6.2. What is chromatic number of C_n ?

Question 6.3. How many edges does C_n have?

7. INDUCED SUBGRAPHS

Definition 7.1. A *induced subgraph* of a graph is a graph contained in a larger graph, such that all the edges connecting the vertices are also contained in the larger graph. For example:



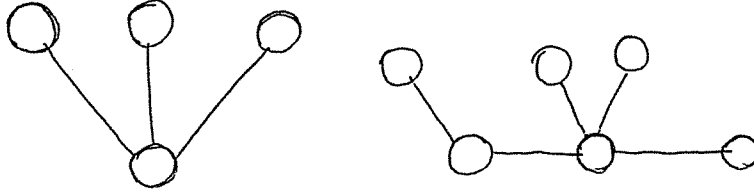
Question 7.2. Show that if the graph G has a induced complete subgraph with n vertices, then chromatic number $\chi(G) \geq n$

Definition 7.3. A induced subgraph which is complete is called a *clique*

Question 7.4. If a graph has a clique with n vertices, is the chromatic number necessarily equal to n ? Explain.

8. TREES

Definition 8.1. A *tree* is a graph which contains no induced cycles (closed induced graph). A *leaf* is a vertex with only one edge. Examples of trees:



Question 8.2. Does every tree have a leaf?

Question 8.3. What is the chromatic number of a tree?

Question 8.4. How many edges does a tree have?