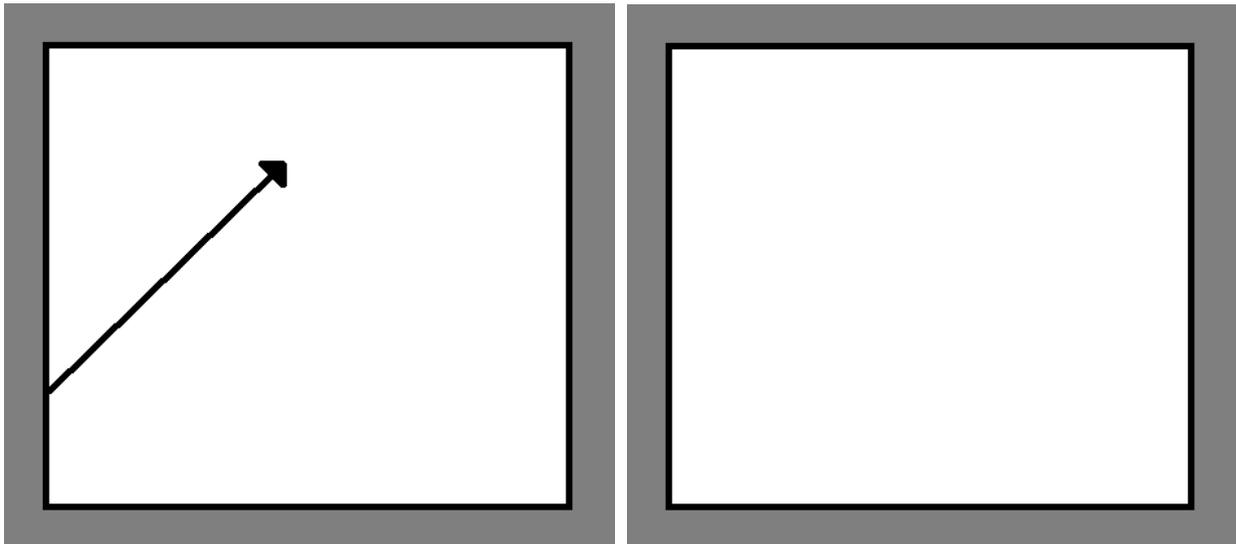


The Mathematics of Billiards: Individual Questions

Washington University Math Circle

March 6, 2016

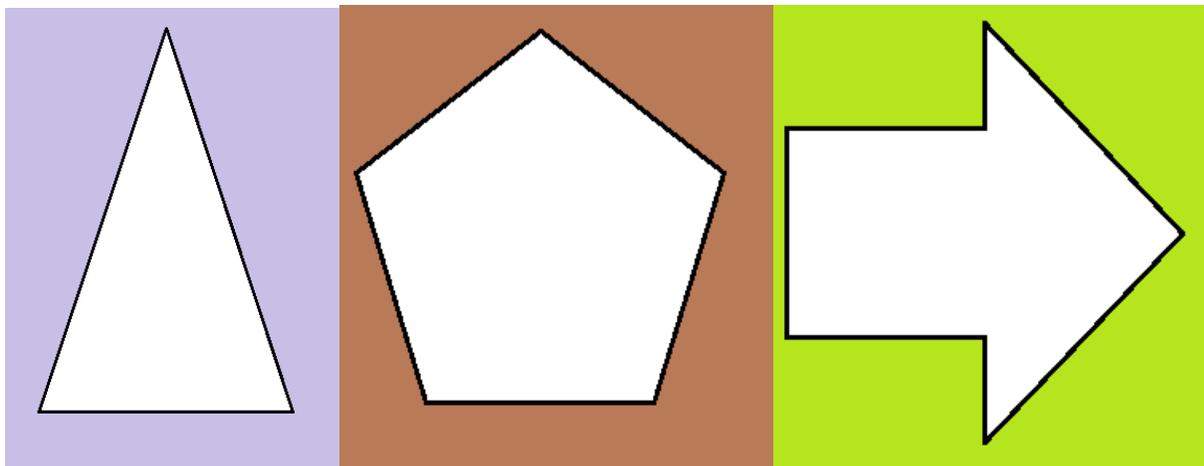
1. **Warm-up:** In the first “table”, sketch the path of the given billiard using the rule that the angle it comes in at is the angle it leaves at. Draw in several collisions. Then for the second table, pick your own starting point and direction and try again.



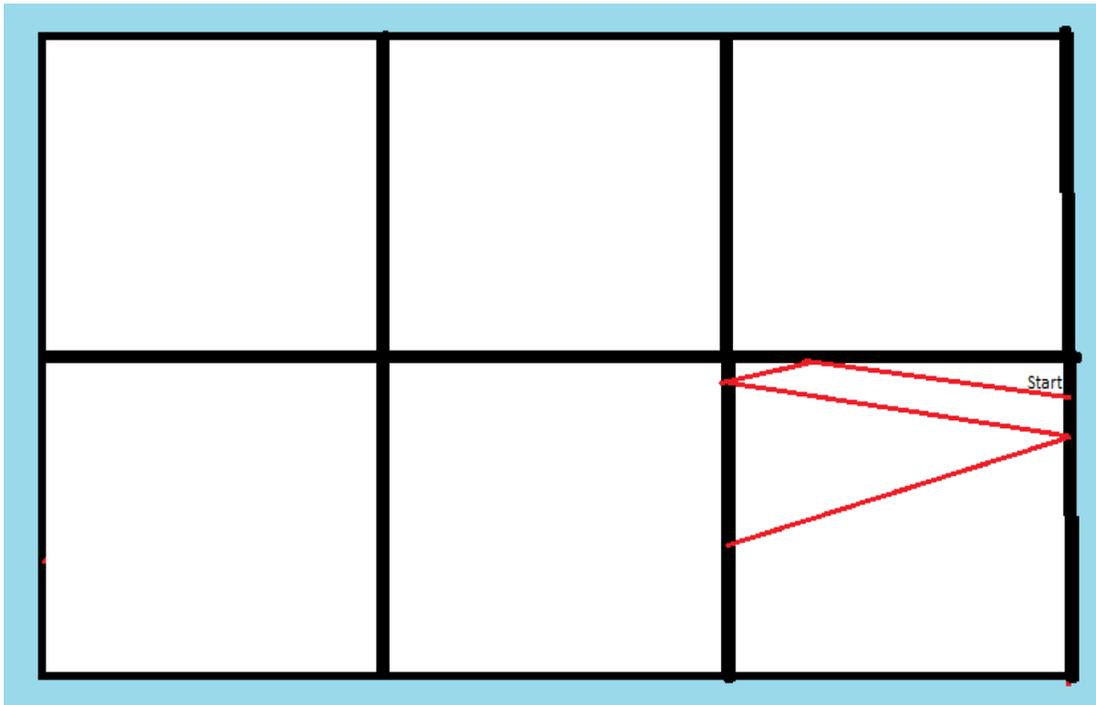
Think about:

- For a rectangle, what can you say about how many different directions the ball will go?
- What would happen if you kept going until you had thousands of collisions? What would the picture look like? Does it depend on where and what direction you decide to start?

2. **For each table below, pick a starting point and direction and sketch in the billiard path.**

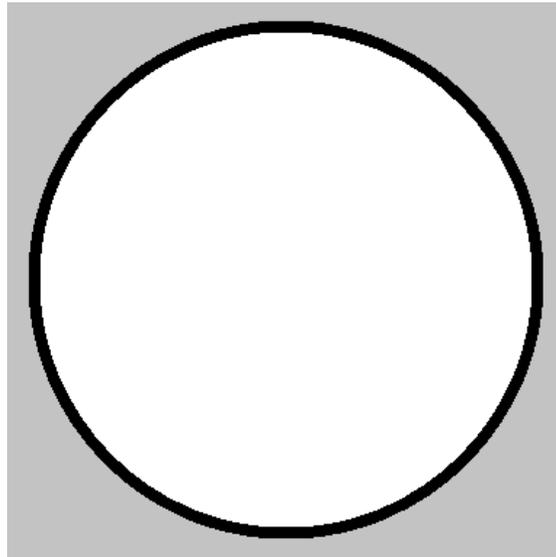
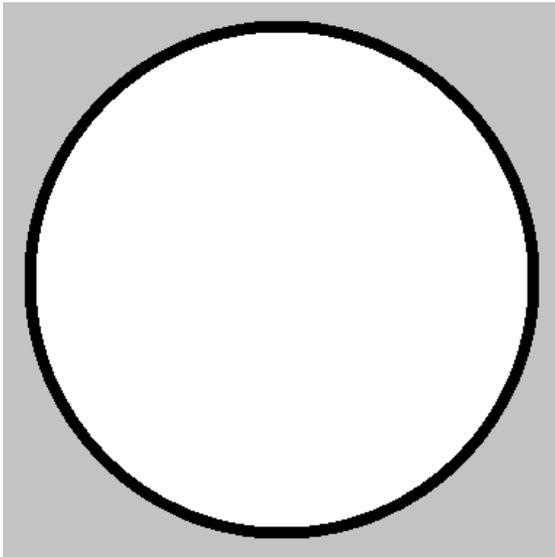


3. Unfold the billiard path.



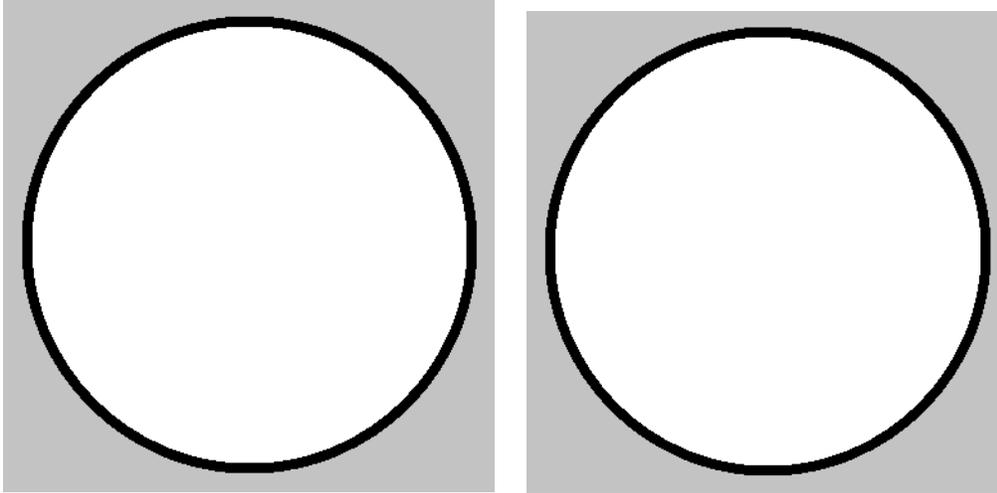
4. What do circular billiards look like?

Pick a starting point and draw the path of the billiard on the circle using the fact that the reflection angle is the same at every collision. Do a lot of paths and try two different starting angles.

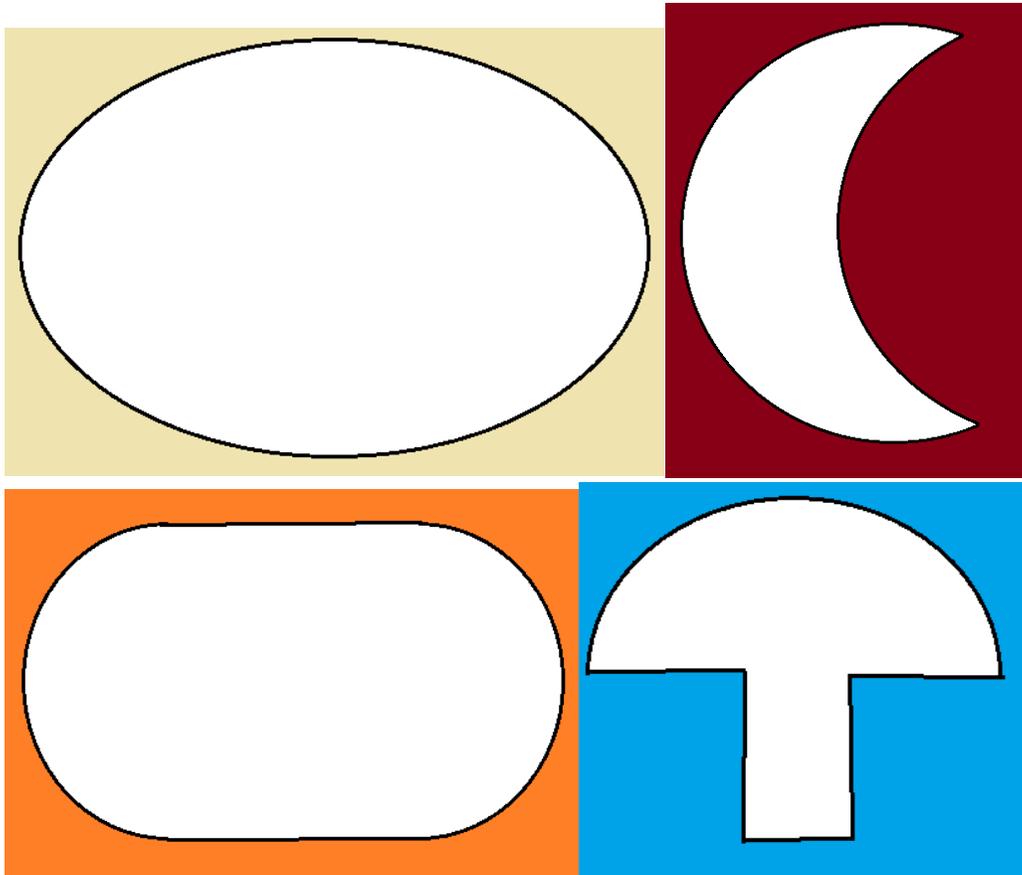


Think about: will the circle ever get completely filled like it did for the moon billiard?

5. A billiard is called “periodic” if it returns to the same point (and direction) after a certain number of collisions. Can you choose directions so that you get periodic billiards on the circle? (Remember: you still need to follow the “specular reflection” rule!)



6. Try some other billiards with curves! (Not necessarily periodic.)



Think about: do these shapes have a pattern like the circle or square or is something more complicated happening?

8. What do no-slip circular billiards look like?

“No-slip” billiards follow a different rule: every other time we switch the angle!

