

Partitions of the Plane and Tilings

Math Circle

September 18, 2016

- (1) What are the angles for the regular polygons?

shape	sides	angle
Equilateral Triangle	3	60°
Square	4	
Regular Pentagon	5	
Regular Hexagon	6	
Regular Heptagon	7	
Regular Octagon	8	

- (2) Check the first few regular polygons and determine which ones fit together to give a partition of the plane. Draw sketches of what it looks like trying to fit the polygon together at a point, and for the cases where they fit together sketch the partition for the plane.

- (3) Of the regular polygon partitions, which one is most efficient (in terms of enclosing the area with the least amount of perimeter)?

shape	sides	perimeter (unit area)
triangle	3	
square	4	
hexagon	6	
circle	curved	≈ 3.54

- (4) Find a partition that uses two or three (or maybe four, but no more!) polygons to partition the plane into regions. Or can you find a partition that uses one non-regular polygon?

(5) Can you create a partition entirely from circles? Try it! What goes wrong?

(6) Can you create a partition entirely from arcs of circles?

(7) What is the best way to partition the sphere into two equal regions? Three? Four?

- (8) Create a pentagonal tiling. You must use only pentagons, but they don't have to be regular.
- (9) Group project: using the templates for the two types of pentagons, create a Cairo-Prismatic tiling. You might wish to color or otherwise mark your tiling to that it is clear which pentagons are which type. Remember: there should be no empty space! After you finish one you might investigate to see if you have a new one or one of the known types.