

Geometric Numbers

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1 Triangle Numbers

Triangular numbers are numbers that can be arranged in a triangular pattern. Draw a triangle below.

The n -th triangular number T_n is formed using an outer triangle whose sides have n dots. Set $T_1 = 1$.

1. Find T_2, T_3, T_4, T_5 .

2. Can you find a pattern for T_n ? Prove your pattern.

3. Compute $T_1 + T_2$, $T_2 + T_3$, $T_3 + T_4$.

4. Can you find a pattern for $T_n + T_{n+1}$? Prove your patten.

5. Can you find a formula relating T_{2+3} to T_2 and T_3 ?

6. Can you find a formula relating T_{a+b} to T_a and T_b ?

7. Can you find a formula for T_{ab} ?

2 Square Numbers

Square numbers are numbers that can be arranged in a square pattern. Draw a square below.

The n -th square number S_n is formed using an outer square whose sides have n dots. Set $S_1 = 1$.

1. Find S_2, S_3, S_4, S_5 .

2. Can find a patter for S_n ? Prove your pattern.

3. Can you find a geometric proof for the following equation

$$T_n + T_{n-1} = S_n?$$

4. Can you find a triangle number that is also a square number?

5. Can you write 17 in terms of the sum of three square numbers?

6. Can you write 15 in terms of the sum of three square numbers? four square numbers?

7. Prove that any positive number can be written as the sum of at most four square numbers.

3 Pentagon and Hexagon

A pentagonal number is a number that can be arranged in the shape of a pentagon. The n -th pentagonal number P_n is formed using an outer pentagon whose sides have n -dots. Set $P_1 = 1$. Draw a pentagon.

A hexagonal number is a number that can be arranged in the shape of a hexagon. The n -th hexagonal number H_n is formed using an outer hexagon whose sides have n -dots. Set $H_1 = 1$. Draw a hexagon.

1. Find P_2, P_3, P_4, P_5 .

2. Find H_2, H_3, H_4, H_5 .

3. Find a formula for the n -th pentagonal number P_n . Prove the formula.

4. Find a formula for the n -th hexagonal number H_n . Prove the formula.

5. Show that every hexagonal number is a triangular number.

6. Is every triangular number a hexagonal number?

7. How do pentagonal numbers related to triangular numbers? Find a number c such that the following is true: If P is a pentagonal number, then there is a triangular number T such that $P = cT$.

4 Tetrahedral number

A tetrahedral number is a number corresponding to a configuration of points that form a pyramid with a triangular base.

1. Find the first 5 tetrahedral numbers?

2. Find the general formula for the n -th tetrahedral number.

3. Find a formula for $T_n + T_{n-1}$.

4. Are there numbers that are both triangular and tetrahedral?

5. Are there numbers that are both tetrahedral and square?

6. Can you define a square pyramidal number? a pentagonal pyramidal number? formulas for them?

7. Can you find a number that cannot be written as the sum of 4 or fewer tetrahedral numbers?

Pollock's conjecture: Every positive integer is the sum of at most five tetrahedral numbers.