Here are some complete graphs.

1. How many $K_3$ do you see inside $K_4$?

2. How many $K_4$ do you see inside $K_6$?
3. Here I have colored the edges of $K_5$ with two colors red and blue (in black-white printing, red edges are dashed and blue edges are solid) such that no monochromatic triangle appears.

Can you do the same with $K_6$?
4. Color the edges of $K_6$ with two colors say red and blue such that no red $K_3$ and no blue $K_4$ appear.

5. Color the edges of $K_7$ with two colors say red and blue such that no red $K_3$ and no blue $K_4$ appear.
6. Color the edges of $K_8$ with two colors say red and blue such that no red $K_3$ and no blue $K_4$ appear.
7. Can you color the edges of $K_9$ with two colors say red and blue such that no red $K_3$ and no blue $K_4$ appear?

8. Can you show that this is impossible?
9. Find the largest complete graph you can with a coloring of its edges with three colors say red, blue and yellow such that no monochromatic triangle appear.\textsuperscript{1}

\textsuperscript{1}World record is $K_{16}$. 
We would say that 4, 8, 12 form a 3-term arithmetic progression, because the distance between 4 and 8 equals the distance between 8 and 12.

10. Which of the following sequences are in arithmetic progression?

5, 14, 23

5, 14, 23, 33

5, 14, 23, 32

11. Here is a coloring of integers from 1 to 8 with two colors red and blue such that no monochromatic 3-term arithmetic progression appears.

Either do the same for integers from 1 to 9, or prove that this is impossible.
12. Color integers from 1 to 15 with three colors such that no monochromatic 3-term arithmetic progression appears.

13. Color integers from 1 to 20 with three colors such that no monochromatic 3-term arithmetic progression appears.²

²World record is 26.
14. Try to find your largest integer $N$, with a cloring of integers from 1 to $N$ with two colors such that no monochromatic 4-term arithmetic progression appears.\textsuperscript{3}

\textsuperscript{3}World record is 34.
This mathematics is called *Ramsey Theory*, with slogan

**TOTAL DISORDER IS IMPOSSIBLE.**

More grown-up, or even now!, you could follow it from the book: