

Math Circle Questions

1. Consider the following pyramid of numbers

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      1
     2 3 4
    5 6 7 8 9
      :
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What is the middle number of the 40th row?

2. There are 1000 lockers in a school hallway, and there are 1000 students in the school. All of the lockers are closed. The first student runs by the lockers and opens every single one. The second student runs by and closes every second locker. The third student runs by and changes every third locker (opens it if it is closed and closes it if it is open). The fourth student changes every fourth locker and so on for all 1000 students. Which lockers remain open at the end?

3. What is the smallest integer with exactly 11 positive divisors?

4. Four players agreed that after each game the loser must use his own money to double all of the other players' money. They played four games and each of them lost exactly once. At the end they all had \$16 dollars. How much did they all have at the beginning?

5. A Teacher walks into class and writes a number on the board. A student gets up and says, "that number is divisible by 31," the next gets up and says, "that number is divisible by 30," the last student gets up and says, "that number is divisible by 2." The teacher says, "you are all correct except two of you that were wrong in a row." How is this possible? That is, can you find a number divisible by all numbers between 2 and 31 except for a pair of consecutive numbers between them?

6. Find 10 numbers in a row which are composite (not prime)

7. Find a number n such that (n/m) has a remainder of 1 when $m=2,3,4,5,6,7,8, or 9 (harder: Can you find one that is less than 3000?)$

8. Fill a 4x4 Magic Square with numbers that satisfy the following properties:

- the product of numbers in each row is less than 10
- the product of each column is larger than 20

9. There are 4 people standing on one end of a bridge. The first person will take 1 min. to cross the bridge, the second will take 2 min. to cross the bridge the third takes 5 min. to cross the bridge and the last person takes 10 min. to cross the bridge. However, it is dark out and they only have one flashlight. Two people can cross the bridge at a time without it collapsing, and the faster person always slows down to the pace of the slower person so they can walk together with the flashlight. Someone will also have to bring the flashlight back. They will have to make several trips as follows:

2 people cross with the flashlight
1 person brings the flashlight back
2 people cross
1 person brings the flashlight back
2 people cross

What is the minimum time it takes to get everyone to the other side (hint: you can do it in less than 19 min.)

10. 10 humans are abducted by aliens; each represents 10% of the entire human population. The aliens give each abductee either a purple hat or a green hat. The 10 are lined up in a single file line, each facing forward, such that the last person can see the remaining 9's hats, the second to last person can see the remaining 8's hats and so on. No one can see his or her own hat.

The aliens then proceed, starting from the last person, to ask each of the abductees what the color of their hat is. If they guess correctly, they and the 10% of the human population they represent survives; if not, the opposite happens.

Assuming the abductees are given a chance to develop a strategy before they are lined up and questioned: what is the optimal strategy they can utilize (i.e. the one with the highest expected number of survivals)?

During the questioning, the abductees are not allowed to say anything besides their guess for the color of their hat when it is their turn.

11. You are given a set of 9 gold coins but are told that one of the coins is a fake that is lighter than all of the rest. The only tool you have is a balance that tells you when two sides have the same weight or that one side is heavier than the other. Can you tell which coin is fake by only using the balance 2 times?