DISTRIBUTION OF MISSING CARDS – A GOOD ESTIMATE

1. Neatly complete the first eight rows of Pascal’s Triangle:

<table>
<thead>
<tr>
<th>ROW #</th>
<th>PASCAL’S TRIANGLE</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1 1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1 2 1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1 3 3 1</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>1 4 6 4 1</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>1 5 10 10 5 1</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>1 6 15 20 15 6 1</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>1 7 21 35 35 21 7 1</td>
<td>128</td>
</tr>
</tbody>
</table>

2. A family has 5 children. What is the probability that they have:
   A. Exactly three girls? 5/16
   B. At least two boys? 13/16
   C. 4G, 1B or 1G, 4B? 5/16

3. You flip one fair coin six times. What is the probability that you flip:
   A. 3 Heads and 3 Tails? 5/16
   B. 4H, 2T or 2H, 4T? 15/32
   C. All heads or all tails? 1/32

4. On this grid, you can only travel on the gridlines and only East (E) and North (N).
   A. How many different paths are there from START to A?
      4 blocks: 2 E’s, 2 N’s; 6 paths
   B. How many different paths are there from START to B?
      7 blocks: 4 E’s; 3 N’s: 35 paths
   C. BONUS: What is the probability that a path from START to B passes through A?
      6*3/35 = 18/35

Pascal’s Triangle provides exact answers to Questions #2-4, above, but only estimates for the following questions.

5. Dummy and you have a total of 8 Spades. Estimate the probability that the other five Spades are divided:
   A. 2-3 or 3-2? 5/8
   B. 1-4 or 4-1? 5/16
   C. 0-5 or 5-0? 1/16

6. Dummy and you have a total of 9 Hearts. Estimate the probability that the other four Hearts are divided:
   A. 2-2? 3/8
   B. 1-3 or 3-1? 1/2
   C. 0-4 or 4-0? 1/8

7. Dummy and you have a total of 8 Diamonds, missing the Jack, 10, 9, 8, and 2. You lead the Ace, King, and Queen of Diamonds. Estimate the probability that you take all the Diamond tricks. ______
   The Diamonds must split 3-2 or 2-3: (10+10)/32 = 5/8

8. Dummy and you have a total of 7 Clubs, missing the Jack, 10, 9, 8, 5, and 2. You lead the Ace, King, and Queen of Clubs. Estimate the probability that you take all the Club tricks. ______
   The Clubs must split 3-3: 20/64 = 5/16
**PASCAL’S TRIANGLE AS PERCENTS – STILL A GOOD ESTIMATE**

It is often more convenient to express probabilities as percentages rather than as fractions. Of course, then each row must sum to 100%. Complete the first eight rows of Pascal’s Percent Triangle. When needed, round to the nearest half of a percent. Due to rounding, some of your row sums will not be exactly 100%.

<table>
<thead>
<tr>
<th># of Cards</th>
<th><strong>PASCAL’S TRIANGLE as PERCENTS</strong></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>50 50</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>25 50 25</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>12.5 37.5 37.5 12.5</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>6 25 37.5 25 6</td>
<td>99.5</td>
</tr>
<tr>
<td>5</td>
<td>3 15.5 31 31 15.5 3</td>
<td>99</td>
</tr>
<tr>
<td>6</td>
<td>1.5 9 23 31 23 9 1.5</td>
<td>98</td>
</tr>
<tr>
<td>7</td>
<td>1 5 16 27 27 16 5 1</td>
<td>98</td>
</tr>
</tbody>
</table>

1. If Dummy and you are missing six cards in a suit, estimate the percent probability that they split:

   A. 3-3? 31%  
   B. 4-2 or 2-4? 46%  
   C. neither A nor B? 21% [or 23%]

2. Dummy and you have 10 Spades, missing the Queen, 4, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Spade tricks. ________________

   They must split 1-2 or 2-1:  $2 \times 37.5 = 75\%$

3. Dummy and you have 9 Diamonds, missing the Queen, Jack, 7, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Diamond tricks. ________________

   They must split 2-2:  $37.5\%$

4. Dummy and you have 8 Hearts, missing the Queen, Jack, 6, 3, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Heart tricks. ________________

   The only way is if the Q and J are a “doubleton” which can occur in two ways [E or W]:  $2/32 = 6.125\%$

5. Dummy and you have 9 Diamonds, missing the Queen, 7, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Diamond tricks. ________________

   They must split 2-2 OR the Queen must be a “singleton” which can occur in two ways:  $6/16 + 2/16 = 50\%$
CARD DISTRIBUTIONS – THE EXACT PROBABILITIES

For independent events such as boys/girls in a family [or flipping a fair coin], the probability of the next child [or coin] being “Girl” [or “tail”] remains 50% and is not dependent on the gender of the previous child [or result of previous coin flip]. Because of that, Pascal’s Triangle provides their exact distributions and probabilities. However, the probability that the “next” card is a Heart does change based on whether the previous cards were or were not Hearts. Each probability IS dependent on previous cards. Therefore the distribution of cards is a dependent event and Pascal’s Triangle only provides (good) estimates of the distributions of the cards.

EXAMPLE

Dummy and you have 8 Hearts. According to Pascal’s Triangle, the probability that West has 3 Hearts and East has 2 Hearts is approximately 10/32 = \[ \frac{31}{128} \approx 24.25\% \]. Now let’s compute the exact probability.

West and East have a total of 26 cards of which 5 are Hearts and 21 are not Hearts. Let’s calculate the probability that your West opponent has exactly 3 Hearts.

The total number of different hands West could have is \( \binom{26}{13} = 10,400,600 \). The number of West hands with exactly 3 Hearts is \( \binom{5}{3} \cdot \binom{21}{10} = 3,527,160 \). Probability = \( \binom{5}{3} \cdot \binom{21}{10} / \binom{26}{13} = 33.9\% \) [about 2.7% higher]

EXACT Probabilities versus “Pascal Triangle Estimates”

6A. If East and West have 5 Hearts, use the Table from page 2 to compute the estimated probability that they are split:

- 3-2 or 2-3: \( 2 \cdot 31\% = 62\% \)
- 4-1 or 1-4: \( 2 \cdot 15.5\% = 31\% \)
- 5-0 or 0-5: \( 2 \cdot 3\% = 6\% \)

6B. Use the ‘combination method’ [above] to compute these exact probabilities:

- 3-2 or 2-3: \( 2 \cdot 33.9\% = 67.8\% \) {about 5% higher}
- 4-1 or 1-4: \( 2 \cdot \binom{5}{1} \cdot \binom{21}{12} / \binom{26}{13} = 28.3\% \) {about 3% lower}
- 5-0 or 0-5: \( 2 \cdot \binom{5}{0} \cdot \binom{21}{13} / \binom{26}{13} = 3.9\% \) {about 2% lower}