Professional Development

Wonders of Maths

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The School of Aspiration, Inspiration, and Exploration of the Mind
Beautiful Dance Moves

\[ \sin(x) \quad \cos(x) \quad \tan(x) \quad \cot(x) \]

\[ |x| \quad x \quad x^2 \quad x^2 + y^2 \]

\[ \sqrt{x} \quad \sqrt{-x} \quad \frac{1}{x} \quad \text{crap.} \]
The triangle ABC is a right triangle with $\angle B = 90^\circ$. The length $AC = 10$ with the altitude $BD = 6$. Find the area of the triangle.
Do you know how to find the area of a triangle?

Find the area of the equilateral triangle if the perpendicular distances from the point P to the sides of the triangle are 3, 4, 5.
It brings them home!

Campbells' Soups
Home Plate for the Little League

$12^2 + 12^2 = 288$

$17^2 = 289$
math
magic
Each design on this page is really half of a word. To obtain the complete word, you have to apply mathematical principles of rotation, reflection (chirality), symmetry, balance, & inversion. This is a good practice on learning ambigrams.
Fields of Mathematics

Mathematical Logic
Set Theory
Category Theory
Theory of Computation
Combinatorics
Number Theory
Group Theory
Graph Theory
Order Theory
Algebra
Boolean Algebra
Geometry
Trigonometry
Calculus
Differential Geometry
Topology
Fractal Geometry
Measure Theory
Vector Calculus
Differential Equations
Dynamical Systems
Chaos Theory
Knot Theory
Complex Analysis
Mathematical Physics
Fluid dynamics
Numerical Analysis
Optimization
Probability Theory
Statistics

Cryptography
Mathematical Finance
Game Theory
Mathematical Biology
Mathematical Chemistry
Mathematical Economics
Control Theory
(David) Crockett Johnson  National Museum of American History

Johnson & his wife Krauss inspired Maurice Sendak, Charles Schultz, & many others.
Who are they? Match letters & numbers
1. Ralph Abernathy
2. Maylm Blalik
3. Harry Blackmun
4. Lewis Carroll
5. David Dinkins
6. Alberto Fujimori
7. Art Garfunkel
8. Grace Hopper
9. Hedy Lamarr
10. Danica McKellar
11. Edwin Moses
12. Florence Nightingale
13. Natalie Portman
14. Sally Ride
15. David Robinson
16. Alexander Solzhenitsyn
17. Bram Stoker
18. Virginia Wade
a. first woman Rear Admiral
b. author of Dracula
c. Nobel Prize winner
d. basketball star
e. Oscar winning actress
f. first American woman in space
g. pioneer in nursing
h. 3-time Olympic champion
i. actress and the theorem named after her
j. actress, invented and patented “frequency hopping”
k. Wimbledon Champion
l. civil rights leader
m. former president of Peru
n. former mayor of New York
o. Charles Dodgson
p. former Supreme Court Justice
q. actress on the Big Bang Theory
r. musician
Wake-Up Your Mind Problems

Given two integers 2 and 3, how many different valued numbers can you create? No math symbols are allowed. Look at the examples below. Can you think of more?

There are 9 overlapping triangles formed with three lines. Can you create 9 non-overlapping triangles using three lines?

23

32

$2^3$

$3^2$
Kobon triangle problem

It is an unsolved problem in combinatorial geometry first stated by Kobon Fujimura. The problem asks for the largest number $N(k)$ of nonoverlapping triangles whose sides lie on an arrangement of $k$ lines.
How many triangles can you find? What is the digital root of the correct number of triangles?
1. **M** 1931 German drama-thriller film
   Rotten Tomatoes 100%

2. **M** for Mersenne Numbers
   $M_p$ for Mersenne Prime... a Mersenne prime is a prime number that is one less than a power of two. It is a prime number of the form $M_n = 2^n - 1$ for some integer $n$. On January 25, 2013, Curtis Cooper, a mathematician at the University of Central Missouri, discovered a 48th Mersenne prime, $2^{57,885,161} - 1$. On January 19, 2016, Cooper published his discovery of a 49th Mersenne prime, $2^{74,207,281} - 1$. The largest is $M_{50}$ GIMPS

3. American Sign language
   Braille for M numeral
   Morse Code for M — —
   ASCII M = 77 m = 109
Tetration
Superexponentiation
Introduced by Goldstein 1947
Standard notation or Knutte notation

\[ ^n a = a \uparrow \uparrow n \]

\[ ^3 5 = 5 \uparrow \uparrow 3 = 5 \]
Catalan’s Conjecture
1844
\[ x^a - y^b = 1 \]
for \( a, b > 1, \ x, y > 0 \)
powers are consecutive &
the difference between two
numbers is 1, then the only
solution is \( x = 3, \ a = 2, \ y = 2, \ b = 3 \)
\[ 3^2 - 2^3 = 1 \]

Catalan's conjecture was proven by Preda Mihăilescu
in April 2002.
Fermat Last Theorem

Fermat claimed to have discovered a proof that the Diophantine equation

\[ x^n + y^n = z^n \]

has no integer solutions for \( n > 2 \) and \( x, y, z \neq 0 \).

Pythagoras had infinitely many solutions for

\[ x^2 + y^2 = z^2 \]

e.g. \( 3^2 + 4^2 = 5^2 \)

Interestingly we have the following:

\[ 3^3 + 4^3 + 5^3 = 6^3 \]

Can you solve the following?

\[ x^3 + y^3 = z^4 \]

Yes, we do have a solution for this equation.
Euler's Conjecture

Euler's conjecture is closely related to Fermat's Last theorem.* So let's look at Euler's conjecture.

We know that \( a^3 + b^3 + c^3 = d^3 \) has an interesting solution. But Euler in 1769 proposed that there are no sets of numbers such that

\[
\begin{align*}
& a^4 + b^4 + c^4 = d^4 \quad \text{or} \\
& a^5 + b^5 + c^5 + d^5 = e^5 \\
& \text{& so on}
\end{align*}
\]

Here is a kicker.

With a help of computers, mathematicians now disproved by showing counterexamples. One of the greatest math minds like that of Euler lost to the computing power.

\[
\begin{align*}
& 95800^4 + 217519^4 + 414560^4 = 422481^4 \\
& 27^5 + 84^5 + 110^5 + 133^5 = 144^5
\end{align*}
\]

Fermat's Last Theorem presentation - Maths is Good for You

http://www.mathsisgoodforyou.com/presentations/fermatstheorem.ppt
B. Practice
   Practice
   Practice
   Practice makes perfect.

C. Evaluation
   Look for Applications
   Review

D. Recreational Maths & Challenges
   Perspiration, Persistence, Perseverance,

A. Intro Concepts & Theorems
   Show & Tell
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How do you make them equal without changing digits?

100 = 102
103 = 112
#10 = #7
560 = 600
0 = 1
10 + 4 = 2
1 + 1 = 10
1 + 1 = 0
1 + 1 = 1
The Power of Mathematical Symbols

You can use any amount of symbols but you have to come up with a valid equation for all of them.

\[
\begin{align*}
000 &= 6 & ? \\
111 &= 6 & ? \\
222 &= 6 & 2 + 2 + 2 = 6 \\
333 &= 6 & 3 \times 3 - 3 = 6 \\
444 &= 6 & ? \\
555 &= 6 & 5 + 5 / 5 = 6 \\
666 &= 6 & 6 + 6 - 6 = 6 \\
777 &= 6 & 7 - 7 / 7 = 6 \\
888 &= 6 & ? \\
999 &= 6 & ?
\end{align*}
\]
Math Symbols Challenge ...using three 3s & math operations, make them equal to the following: (try not to use $3/3 = \sqrt{3}/\sqrt{3} = 3!/3! = .3/3 = 1$ for the same problem)

0 = \frac{(3 - 3)}{3} = (3 - 3)^3 = \underline{} 
1 = 3^{3-3} = (\frac{3}{3})^3 = \frac{\sqrt{3}\sqrt{3}}{3} = \underline{} 
2 = \frac{(3 + 3)}{3} = 3 - \frac{3}{3} = \underline{} 
3 = 3 + 3 - 3 = 3 \cdot \left(\frac{3}{3}\right) = \underline{} 
4 = 3 + \frac{3}{3} = 3 + (3 - 3)! = \underline{} 
5 = 3! - \frac{3}{3} = \frac{3!}{3} + 3 = \underline{} 
6 = 3 \times 3 - 3 = \underline{} 
7 = (\frac{3}{3}) - 3 = \underline{} 
8 = 3! + \frac{3!}{3} = \underline{} 
9 = 3 + 3 + 3 = 3\sqrt{3}\sqrt{3} = \underline{} 
10 = challenge: Find 10 solutions that are equal to 10.
1) Which is heavier
   1 oz of gold or 1 oz of iron ore?

2) Which is heavier
   1 pound of cotton or 1 pound of gold?

3) At what temperature
   Fahrenheit and Celsius scales
   intersect?

4) What kind of crazy numbers
   are they?  BAD  BABE
   DEAD  DEFECE8  BOOB1E5

5) Which is greater
   .9999999999999999...  or 1?
Which is greater?

.99999999... ___ 1

Method 1

\[
\begin{align*}
1/9 &= .11111111111111111... \\
2/9 &= .22222222222222222... \\
3/9 &= .33333333333333333... \\
8/9 &= .88888888888888888... \\
9/9 &= .99999999999999999...
\end{align*}
\]

Method 2

\[
x = .99999... \quad \text{then} \quad 10x = 9.99999...
\]

\[
10x - x = 9.99999... - .99999... = 9
\]

\[
9x = 9 \quad x = 1 \quad \text{then} \quad .99999... = 1
\]

Method 3

\[
.99999999... = .9 + .09 + .009 + .0009...
\]

sum of infinite geometric series \( s = \frac{a}{1 - r} \)

\[
a = .9 \quad r = .1 \quad \text{therefore} \quad .9/(1 - .1) = .9/.9 = 1
\]
Jason Chaffetz of Utah (R) presented the graph at Congressional Hearing.
...times 35896...71 dollars and 79 cents.
The TV is the center of many households – where family members gather to spend time together and unwind. Whether your tastes veer toward obscure foreign films or reality shows, a quality TV will help bring them to life. Now comes the hard part: which TV should you buy?

**BEST OF THE BEST**

- **Samsung** 75" Class 4K (2160P) Smart OLED TV
- **LG** 65-inch 4K Ultra HD Smart OLED TV
- **Sony** 48-inch 1080p Smart LED TV
- **Samsung** 65" Class 4K (2160P) Smart LED TV
- **TCL** 55-inch 4K Ultra HD Roku Smart LED TV
What is Sabermetrics?

Statistical analysis of baseball
Coined by Bill James
Comes from SABR
Turned mainstream with Moneyball

batting average, Drek Jeeter vs David Justice
on-base percentage,
slugging percentage,
OPS (on-base plus slugging)
\[ \text{OPS} = \frac{\text{HITS} + \text{WALKS}}{\text{TOTAL BASES}} \]
\[ \text{RUNS} = \frac{\text{AT-BATS} + \text{WALKS}}{} \]

\[ \frac{\text{RUNS}}{\text{RUNS}^2} + \frac{\text{WINS}}{\text{WINS} + \text{LOSSES}} = \frac{\text{RUNS}^2}{\text{RUNS}^2 + \text{OPPOSITION} \text{ RUNS}^2} \]

Inning pitched \[ 5.2 + 6.1 = 12 \]
\[ \frac{1}{3} + \frac{1}{5} = \frac{2}{8} \]

Mediant Fractions
What is Simpson's Paradox?

\[ \frac{a}{b} < \frac{A}{B} \]
\[ \frac{c}{d} < \frac{C}{D} \]

But this can happen.

\[ \frac{(a+c)}{(b+d)} > \frac{(A+C)}{(B+D)} \]

arithmetical illustration

\[ \frac{1}{5} < \frac{2}{8} \]
\[ \frac{6}{8} < \frac{4}{5} \]
\[ \frac{7}{13} > \frac{6}{13} \]
Math Trivia Game II  (the use of smartphones or other electronic devices are not allowed)

1. A spigot algorithm is an algorithm which generates digits of a quantity one at a time without using or requiring previously computed digits. The spigot algorithms are known for two famous numbers. Name both. One of them is sometimes called Ludolph's constant or Archimedes constant?

2. Who proved in 1796 (when he was only 19 years old) that this regular heptadecagon was constructible with a compass and straightedge? He was so proud of his discovery that he wished this figure to be carved on his tombstone.

3. What is a flat image that can be displayed in 3 dimensions?

4. What number does “giga” stand for?

5. The smallest unit for “binary digit” is called the bit. (0 & 1). Then two bits are called _________ four bits are called _________ eight bits are called a _________

6. What is unique about 8549176320? It is divisible by all the digits except by 7.

7. What number, a one followed by 100 zeros was first used by nine-year-old Milton Sirotta in 1940?

8. How many faces does a icosahedron have?

9. What two letters are both symbols for 1,000?

10. What handy math instrument's days were numbered when the pocket calculator made the scene in the 1970s?

11. What is the name of the famous, controversial series 1 - 1 + 1 - 1 + 1…?

12. Arithmetic Average: (a+b)/2  Geometric Average: \(\sqrt{ab}\)

    Then what expression is used for the Harmonic Average in terms of a & b?

13. According to Schlafli symbol, what does \{4,3\} represent?

14. Archimedes discovered the ratio of the volume of a sphere (internally tangent) to the volume of cylinder, He was surprised to find out the ratio was the same when surface areas were compared. He was so impressed with this finding, he wished his tombstone to be engraved with the cylinder and the internally tangent sphere. What was the ratio?

15. At what temperature Celsius and Fahrenheit scales meet the same number?

16. What is Jenny's phone number (prime number) according to Tommy Tutone?

17. What does 4:20 on 4/20 for 420-ing mean?

18. What is the word that caused frequent quarrels and controversies among 17th-century mathematicians and it was often referred to as “The Helen of Geometers”. Brachistochrone & tautochrone are associated with this curve.

19. Euler's equation: \(V - E + F = 2\). What does each letter stand for?

20. What number is common to both 3x3 magic square sum & the game falsely claimed by famous puzzle-maker Sam Loyd he invented the game?

21. 0 1 1 2 3 5 8 13... is called Fibonacci numbers sequence, then what do you call the following sequence? 0 1 2 5 12 29 70 169 ...

22. What does TSP stand for in the mathematical field of optimization?

23. What letter do electrical engineers use in place of the imaginary number i?
Find the missing number in the following sequence:

10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24, __, 100, 121, 10000
1. Sicherman’s Difference Triangles if 1-6, then

   How about 1-10 or 1-15?

2. Prime Neighbors sum 1-16
   There are many of them.
   10, 1, 2, 11, 12, 5, 8, 3, 16, 13, 6, 7, 4, 9, 14, 15

3. Square sum 1-17
   e.g. 9 - 7 - 2 - 14 - and so on

4. Magic Hexagon 1-19 sum=38

5. Graceful Graphs & Graceful Tree Conjecture

6. I can guess your number!

7. same 4 numbers in a square
1) Prime Pair Sum 1-16
2, 1, 10, 3, 16, 13, 6,...

if you start with the following numbers instead of 2, what would be answer for 1 or 3, or 4, or 5...

2) Square Sum 1-16
8, 1, 15, 10, 6, 3, 13, 12, ... can you start with other other numbers?

3) Graceful Tree Conjecture
Isaac Newton’s tree-planting puzzle

1. How can nine trees be arranged in ten rows, such that each row contains exactly three trees?

2. You have 10 trees. How do you plant them in five rows with four trees in each row?
Isaac Newton’s tree-planting puzzle

How can nine trees be arranged in ten rows, such that each row contains exactly three trees?

You have 10 trees. How do you plant them in five rows with four trees in each row?
Rebuses

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ping WILLOW  SPACE  SYMPHON  TRAVEL  CCCCCC
BAEDUMR  TT TT TT TT TT DNA4TH

1/25/2006 9:59 AM
\sqrt{625} = 25 \text{ then what is } dr(625) ?

Formula:
\[ dr(n) = n - 9\lfloor (n - 1)/9 \rfloor \]
\[ \lfloor x \rfloor = \text{ floor function} \]

\[ dr(625) = 625 - 9\lfloor (625 - 1)/9 \rfloor = 625 - 9\lfloor (624)/9 \rfloor = 625 - 9\lfloor (69.333...) \rfloor = 625 - 9\lfloor (69) \rfloor = 625 - 621 = 4 \]
so \( dr(625) = 4 \)

Find \( dr(89) = \)
\[
dr(n) = n - 9 \lfloor \frac{(n-1)}{9} \rfloor
\]
digital roots multiplication

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Vedic Math

squared the two digit numbers that end in 5

\[35^2 = 1225 \quad 75^2 = ?\]

Squaring the two digit number starting with 5

\[52^2 = 2704 \quad 56^2 = ?\]

Multiplying two digit numbers where the first figures are the same and the last figures add up to 10

\[32 \times 38 = 1216\]

\[43 \times 47 = ?\]

Multiplying a number by 11

\[26 \times 11 = 281\]

\[12543 \times 11 = ?\]

Multiplying a number by 111

\[123 \times 111 = 13653\]

\[143 \times 111 = ?\]

Multiplying two numbers near 100 but both below 100

\[88 \times 98 = 8624\]

\[\begin{array}{c}
88 \\
12
\end{array}\]

\[\begin{array}{c}
98 \\
2
\end{array}\]

\[\begin{array}{c}
86 \\
24
\end{array}\]
What is so startling about the following word?

Startling
What is so startling about the word **Startling**?

**Startling**

**Starting**

**Staring**

**String**

**Sting**

**Sing**

**Sin**

**In**

**i**
Levenshtein distance

In information theory, linguistics and computer science, the Levenshtein distance is a string metric for measuring the difference between two sequences. Informally, the Levenshtein distance between two words is the minimum number of single-character edits (insertions, deletions or substitutions) required to change one word into the other. It is named after the Soviet mathematician Vladimir Levenshtein, who considered this distance in 1965.

See Lee distance, Hamming distance
Truncatable Primes (prime primes)

The largest left-truncatable prime is the 24-digit

357686312646216567629137
57686312646216567629137
7686312646216567629137
686312646216567629137
86312646216567629137
6312646216567629137
312646216567629137
12646216567629137
2646216567629137
64216567629137
46216567629137
6216567629137
216567629137
16567629137
6567629137
567629137
67629137
7629137
629137
29137
9137
137
37
7

The largest right-truncatable prime is the 8-digit 73939133.
You can make puzzles like growing primes.
1) Lee Distance

It is a distance between two strings of equal length. The distance between 1234 and 2353 is $1 + 1 + 2 + 1 = 5$. The distance between abcd and code is $2 + 13 + 1 + 1 = 17$.

2) Hamming Distance

The Hamming distance between two strings of equal length is the number of positions at which the corresponding symbols are different. In other words, it measures the minimum number of substitutions required to change one string into the other. The Hamming distance between:

"karolin" and "kathrin" is 3. "karolin" and "kerstin" is 3.
1011101 and 1001001 is 2. 2173896 and 2233796 is 3.

The Hamming distance and Lee distance are used to define some essential notions in coding theory, such as error detecting and error correcting codes. They are used in several disciplines including information theory, coding theory, and cryptography.

3) Levenstein Distance

The distance operations are the deletion, insertion, or substitution of a character in the string. The Levenshtein distance is usually what is meant by "edit distance". Spell check or suggested substitution for words or phrases uses Levenshtein Distance.

- **Insertion** of a single symbol. If $a = uv$, then inserting $x$ produces $uxv$.
- **Deletion** of a single symbol changes $uxv$ to $uv$.
- **Substitution** of a single symbol $x$ for a symbol $y \neq x$ changes $uxv$ to $uyv$.

The Levenshtein distance between "kitten" and "sitting" is 3. A minimal edit script that transforms the former into the latter is:

- kitten → sitten (substitution of "s" for "k")
- sitten → sittin (substitution of "i" for "e")
- sittin → sitting (insertion of "g" at the end).
Möbius Strip

Universal recycling symbol designed by Gary Anderson (23-year old college student at USC)

The one-sided surface was discovered by German mathematician August Ferdinand Möbius.

Conveyor belt patented by Goodyear.
US Patent # 3991631

*Continuous-loop recording tapes and typewriter ribbons (obsolete)
*Fabric computer printer

The Möbius strip is now an integral part of mathematics, magic, science, art, engineering, literature, postage stamps, and music. Even Möbius beer (now defunct) was produced with the message: “It will keep you going on and on all night long.”
Möbius Strip

Double Möbius Strips

Martin Gardner's Surprise

Two Möbius strips

Möbius Shorts by Gourmanlin
Hexaflexagon

Flexagons are flat models, usually constructed by folding strips of paper, that can be flexed or folded in certain ways to reveal faces besides the two that were originally on the back and front. The discovery of the first flexagon, a trihexaflexagon, is credited to the British student Arthur H. Stone who was studying at Princeton University in the USA in 1939, allegedly while he was playing with the strips he had cut off his note books. Stone's colleagues Bryant Tuckerman, Richard P. Feynman and John W. Tukey became interested in the idea and formed the Princeton Flexagon Committee. Tuckerman worked out a topological method, called the Tuckerman traverse, for revealing all the faces of a flexagon.

Flexagons were introduced to the general public by the recreational mathematician Martin Gardner, writing in 1956 in his inaugural "Mathematical Games" column for Scientific American magazine and later published The "Scientific American" Book of Mathematical Puzzles and Diversions (Simon & Schuster, 1959).

The most entertaining videos are done by Vi Hart. There are 4 parts:
1) Hexaflexagons
2) Hexaflexagons 2
3) Hexaflexagon safety guide
4) Flex Mex
https://www.youtube.com/playlist?list=PLaNzoFtkQ7rvtb5ac9qdi76iNKuqZWQkB3
(71+1)(71-1)=71
Factorials!

1. Christian Kramp 1808
2. $0! = 1!$ proof by pattern
3. Subfactorial... de Montmort number
4. Multifactorial
5. Superfactorial $\text{sf}(n)$ and $n!$ are different
6. Primorial $n\# = \text{prime factorial}$ coined by Dubner
7. $\Gamma(n) = (n-1)!$
8. $\Pi(n) = n!$
9. Pochhammer (rising & falling)
10. Roman Factorial

$$[n]! = \begin{cases} n! & \text{for } n \geq 0 \\ \frac{(-1)^{n+1}}{(n+1)!} & \text{for } n < 0. \end{cases}$$
Roman Factorial

\[ [n!] = \frac{(-1)^{n-1}}{(-n-1)!} \]

\begin{align*}
  n = -1 & \quad 1/0! \\
  n = -2 & \quad -1/1! \\
  n = -3 & \quad 1/2! \\
  n = -4 & \quad -1/3! \\
  n = -5 & \quad -1/4! \\
  n = -6 & \quad -1/5! \\
\end{align*}
Can you find the three-digit number ABC based on the following?

ABC = A! + B! + C!
Distances Galore

1. Proxemics
2. Mahalanobis
3. Hamming
4. Lee
5. Comoving
6. Levenshtein
7. Chevyshev
8. Manhattan*
9. Orthodromic
10. Genetic
11. Hausdorff
12. Collaborative (Erdős #)
Distances, & distances, etc. etc. etc...

1. **Proxemics...** distance between people as they interact: intimate distance, personal distance, social distance, critical distance, flight distance, propinquity (felicific calculus)

2. **Mahalanobis distance...** Mahalanobis' discovery was prompted by the problem of identifying the similarities of skulls based on measurements in 1927. Mahalanobis distance is widely used in cluster analysis and other classification techniques.

3. **Hamming distance...** In information theory, the Hamming distance between two strings of equal length is the number of positions for which the corresponding symbols are different. Put another way, it measures the minimum number of substitutions required to change one into the other, or the number of errors that transformed one string into the other.

   The Hamming distance between:
   
   * 1011101 and 1001001 is 2.
   * "toned" and "roses" is 3.

4. **Lee distance...** The Lee distance is named after C.Y. Lee. It is applied for phase modulation while the Hamming distance is used in case of orthogonal modulation. The Lee distance between 3340 and 2543 is 6. 1+2+0+3=6. This is used in coding theory.

5. **Comoving distance...** In standard cosmology, 'comoving' distance is a distance measure used by cosmologists to define distances between galaxies. Cosmological distance scale is the calibration of measurements of relative distances between extragalactic objects. It is difficult to measure the distances of, but indirect measurements can be devised. **TRIGONOMETRIC PARALLAX...**

   This method rates an A because it is the gold standard for astronomical distances. It is based on measuring two angles and the included side of a triangle formed by 1) the star, 2) the Earth on one side of its orbit, and 3) the Earth six months later on the other side of its orbit. There are many other ways to measure distances.

6. **Levenshtein distance...** In information theory and computer science, the Levenshtein distance is a metric for measuring the amount of difference between two sequences (i.e., the so-called edit distance). The Levenshtein distance between two strings is given by the minimum number of operations needed to transform one string into the other, where an operation is an insertion, deletion, or substitution of a single character. A generalization of the Levenshtein distance allows the transposition of two characters as an operation.

   It is often used in applications that need to determine how similar, or different, two strings are, such as spell checkers.
For example, the Levenshtein distance between "kitten" and "sitting" is 3, since the following three edits change one into the other, and there is no way to do it with fewer than three edits:
1. kitten → sitten (substitution of 's' for 'k')
2. sitten → sittin (substitution of 'i' for 'e')
3. sittin → sitting (insert 'g' at the end).

7. **Chebyshev distance**... In mathematics, Chebyshev distance is a metric defined on a vector space where the distance between two vectors is the greatest of their differences along any coordinate dimension. It is named after Pafnuty Chebyshev. It is also known as chessboard distance, since in the game of chess the minimum number of moves needed by a king to go from one square on a chessboard to another equals the Chebyshev distance between the centers of the squares.

8. **Manhattan distance** (taxicab distance) The name alludes to the grid layout of most streets on the island of Manhattan, which causes the shortest path a car could take between two points in the city to have length equal to the points' distance in taxicab geometry.

9. **Orthodromic distance**... (the great-circle distance, geodesic distance or surface distance) and it is the shortest distance between any two points on the surface of a sphere measured along a path on the surface of the sphere (as opposed to going through the sphere's interior). Because spherical geometry is rather different from ordinary Euclidean geometry, the equations for distance take on a different form. The distance between two points in Euclidean space is the length of a straight line from one point to the other. On the sphere, however, there are no straight lines. In non-Euclidean geometry, straight lines are replaced with Geodesics. Geodesics on the sphere are the great circles. The simplest way to calculate geodesic distance is to find the angle between the two points, and multiply this by the circumference of the earth. (Considering the earth to be spherical rather than an ellipsoid.)

10. **Genetic distance**... Genetic distance is a measure of the dissimilarity of genetic material between different species or individuals of the same species. By comparing the percentage difference between the same genes or junk DNA of different species, a figure can be obtained, which is a measure of "genetic distance". Depending upon the difference, and correcting this for known rates of evolution, genetic distance can be used as a tool to construct cladograms showing the family tree of all living beings. There are several different methods for defining genetic distance. One genetic distance measure is given by the formula $D = -\log(\text{In})$. For instance, the fact that the genetic distance of chimpanzees and human beings is only ~5% (about 95% of the base pairs are exactly shared between human and chimpanzee DNA), suggests that human beings and chimpanzees last had a common ancestor about 5 million years ago, and that chimpanzees and humans are more closely related than either of the two species are related to gorillas and orangutans (which diverged about 9 million years ago, and 12 million years ago, respectively).
The Luhn Algorithm

It is a checksum formula (mod 10) used to validate a variety of identification numbers. It was created by a German IBM scientist Hans Peter Luhn. It is not intended to be a cryptographically secure hash function; it was designed to protect against accidental errors like mistyped incorrect numbers.

do double all the underlined numbers
4 4 3 7 1 2 1 4 5 6 1 8 9 1 7 3
8 6 2 2 10 2 18 14
add all the numbers not underlined and doubled numbers
4 + 7 + 2 + 4 + 6 + 8 + 1 + 3
+ 8 + 6 + 2 + 2 + 1 + 0 + 2
+ 1 + 8 + 1 + 4 = 70

Most credit or debit cards have 16 digits. American Express has only 15 digits. Underline the 2nd digit and every other one and follow the same rule. The Luhn Algorithm detects only any single-digit error. The improvements were made by the Verhoeff algorithm and the Damm algorithm.
The first digits: Visa card = 4, MasterCard = 5 American Express = 3 Gas card = 7
Erase four dots to make a valid equation.

\[(10+11)(10-11)=100+11\]
a valid equation

\[(10 + 1) (10 - 1) = 100 - 1\]
Rich History of i, complex numbers, etc

1. \( i = \sqrt{-1} \quad i^2 = -1 \)
2. complex number \( a + bi \)
3. \( |a + bi| \)
4. Argand Diagram
5. Gaussian integers
6. Quaternions: \( a + bi + cj + dk \)
   \( i^2 = j^2 = k^2 = ijk = -1 \)
7. Octonions
   \( a_0 + i_1 a_1 + i_2 a_2 + i_3 a_3 + i_4 a_4 + i_5 a_5 + i_6 a_6 + i_7 a_7 \)
8. Hurwitz’s Theorem
9. Mandelbrot & complex numbers

challenge questions

a. \( i^i = ? \)
   b. \( \sqrt{i} = ? \)
5 Amazing & Cool Maths

1) Benford’s Law
2) Ulam’s Spiral
3) TSP
4) 17 wallpaper groups
5) $e^{\pi i} + 1 = 0$
Benford’s Law
Log \((1 + 1/D)\)

\(D = \text{digit}\)

A phenomenological law also called the first digit law. Benford’s law states that in listings, tables of statistics, etc., the digit 1 tends to occur with probability \(\sim 30\%\), much greater than the expected 11.1\% (i.e., one digit out of 9).
Ulam Spiral

37 36 35 34 33 32 31
38 17 16 15 14 13 30
39 18 5 4 3 12 29
40 19 6 1 2 11 28
41 20 7 8 9 10 27
42 21 22 23 24 25 26
43 44 45 46 47 48 49 ...

multiples of 3
multiples of 7
multiples of odd integers
Science and Culture: Solving a math problem to create art
by Steve Ornes  2016

Mathematician Robert Bosch never intended to become an artist. But 17 years ago, he started looking for ways to engage his students in optimization research, his field of expertise.

Robert Bosch’s optimization artworks have included a rendering of the Mona Lisa, which he starts by identifying the points or “cities” (Left) through which the Traveling Salesman route will run (Right).

Optimization is the mathematical quest for the best way to do something, from finding the shortest distance between two places to figuring out the best way to pack a suitcase. It often involves calculating the highest or lowest value of something. The applications are far-reaching. To Bosch, they also offered a pleasing aesthetic. “I wanted to convince my students that this material I teach is beautiful and incredibly applicable,” says Bosch, who teaches at Oberlin College in Ohio. “My mission was to show them that pretty much any field you could think of has optimization applications.”

Bosch is now known among mathematicians—and the math–art subset of that community—for his line drawings, mosaics, and sculptures created using solutions to optimization problems. He’s also organized showings of mathematical art at galleries and conferences, and has given lectures to audiences ranging from elementary school children to the mathematically curious at the Museum of Math in New York.
A wallpaper group

(or plane symmetry group) is a mathematical classification of a two-dimensional repetitive pattern, based on the symmetries in the pattern. Such patterns occur frequently in architecture and decorative art, especially in textiles and tiles as well as wallpaper.

Example A and Example B belong to the same wallpaper group; it is called p4m in the IUC notation and *442 in the orbifold notation. Example C has a different wallpaper group, called p4g or 4*2. The fact that A and B have the same wallpaper group means that they have the same symmetries, regardless of details of the designs, whereas Example C has a different set of symmetries despite any superficial similarities.

Different Notations
1) ICU notation...International Union of Crystallography notation by the International Council of Science
2) Orbifold notation: advocated by John Horton Conway, is based not on crystallography, but on topology. We fold the infinite periodic tiling of the plane into its essence, an orbifold, then describe that with a few symbols.
3) Coxeter notation...far more rigorous math explanations of symmetry, pattern

Conway, Coxeter and crystallographic correspondence

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<th>Conway</th>
<th>o</th>
<th>xx</th>
<th>*x</th>
<th>**</th>
<th>632</th>
<th>*632</th>
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<td>[(∞,2)+,∞+]</td>
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<td>pg</td>
<td>cm</td>
<td>pm</td>
<td>p6</td>
<td>p6m</td>
</tr>
</tbody>
</table>

Why there are exactly seventeen groups???

Isometries or symmetries of the Euclidean plane (translations, rotations, reflections, glide reflections) & other criteria are used for determination
1729
Taxi cab number
Ramanujan number

Hardy said it was a dull number, but for Ramanujan it was an interesting number.

\[ 1729 = 9^3 + 10^3 \]
\[ = 1^3 + 12^3 \]
\[ 1 + 7 + 2 + 9 = 19 \]

vertical symmetry of 19 is 91
and when you multiply

\[ 19 \times 91 = 1729 \]

very interesting, no?
Diophantine Equation Problems
using digits 0 - 9

ABC = A^3 + B^3 + C^3
EFG = E^3 + F^3 + G^3
HJK = H^3 + J^3 + K^3
LMN = L^3 + M^3 + N^3

0^3 = 0  1^3 = 1  2^3 = 8  3^3 = 27
4^3 = 64  5^3 = 125  7^3 = 343

153 = 1^3 + 5^3 + 3^3
371 = 3^3 + 7^3 + 1^3
370 = 3^3 + 7^3 + 0^3
407 = 4^3 + 0^3 + 7^3
From time immemorial people were fascinated with three geometric figures: a circle, a square, & a triangle.
Can you identify this?

\[ Q \leq \frac{4\pi A}{p^2} \]
Progressive Challenges
Long ago, someone drew a triangle and three segments across it. Each segment started at a vertex and stopped at the midpoint of the opposite side. The segments met in a point. The person was impressed and repeated the experiment on a different shape of triangle. Again the segments met in a point. The person drew yet a third triangle, very carefully, with the same result. He told his friends. To their surprise and delight, the coincidence worked for them, too. Word spread, and the magic of the three segments was regarded as the work of a higher power.

Centuries passed, and someone proved that the three medians do indeed concur in a point, now called the centroid. The ancients found other points, too, now called the incenter, circumcenter and orthocenter. More centuries passed, more special points were discovered, and a definition of triangle center emerged. Like the definition of continuous function, this definition is satisfied by infinitely many objects, of which only finitely many will ever be published. The Encyclopaedia of Triangle Centers (ETC) extends a list of 400 triangle centers published in the 1998 book Triangle Centers and Central Triangles. For subsequent developments, click Links (one of the buttons atop this page). In particular, Eric Weisstein's MathWorld, covers much of classical and modern triangle geometry, including sketches and references.
Triangular Affairs

You already know triangles are classified according to angles and sides. (e.g. Acute, Obtuse, Right, Equiangular or Scalene, Isosceles, Equilateral). Triangles are one of the most important figures in Geometry and also in real life constructions. There are so many fascinating, interesting, and unusual triangles that are not usually found in our textbooks. So, let's enrich our knowledge on triangles and be an expert on the topic.

1. Archimedian
2. Distress flight pattern
3. Golden
4. Cevian
5. Reuleaux
6. Penrose
7. Heronian
8. Work
9. Kanizsa
10. Morley’s
11. Spherical
12. Circular
13. Pedal
14. Medial (a.k.a. Auxiliary)
15. Yff (no kidding. This is real.)
16. Number
17. Homothetic
18. Tangential
19. Triangle Squaring
20. Orthic
21. Basic Drafting
22. Speed Square
23. Floyd
24. Eutrigon
Reuleaux triangle

It is the curve of a constant width. Because all its diameters are the same, the Reuleaux triangle is one answer to the question "Other than a circle, what shape can a manhole cover be made so that it cannot fall down through the hole?"

An amazing fact: ratio between the length of the entire curve to the width is $\pi$.

1. Reuleaux polygons are used for coins in several countries.
2. Drilling a square hole (a mortise and tenon joint)
3. Some pencils are made in this shape (more comfortable, encourages proper grip and less likely to roll off the table)
4. US National Park System
5. The logo of Colorado School of Mines
6. The logo of The Connecticut Collegiate Mathematics Competition
7. Guitar picks
8. Corporate logo of Petrofina (Belgian Oil Co.)
9. Very close to the rotor of Wankel Engine
10. Valve covers used in the Mission Bay Project of San Francisco to differentiate reclaimed water from portable water are in the shape
11. Leonardo da Vinci created a world map in which the spherical surface of the earth was divided into eight octants, each flattened into the shape of a Reuleaux triangle.
Isomax Foam

Materials scientist Jonathan Berger has developed a solid that he describes as "the most efficient in the universe." Called Isomax, this material—in comparison to other similar engineering materials—has the highest stiffness to lightness ratio, meaning that for its relatively low density, it has the highest stiffness, resisting crushing and shearing forces that would buckle and flatten denser, heavier materials. It also can be easily manufactured and, with fairly minor tweaks, changed to emphasize different properties to function in various ways without sacrificing its structural integrity.

"We were trying to answer the fundamental question of how to add space to stuff," Berger says, of the research behind Isomax.

Void space, he explains, is an aspect that plays an important role in the qualities of virtually every object, such as the functionality of buildings, the taste and texture of foods, or the ability of our shoes to cushion our feet. How that space is utilized, enclosed, or set apart, contributes to the objects’ characteristics such as softness, weight, flexibility, stiffness, and strength.

2 basic shapes: triangle and cross

Key to the innovative technology behind Isomax is an old science: geometry. The foam has an ordered topology of regularly occurring cells featuring two basic shapes—the triangle and the cross.

The intersecting walls of the three-dimensional cross shape are ideal for resisting perpendicular crushing forces, while the pyramidal shapes, long known for their stability, resist shearing forces.

Combined in a repeating pattern, these cells are made to withstand forces from all directions while maintaining the foam’s low density, making it ideal as a structural material. Isomax’s low density for its stiffness means that things made from it require less material for the desired amount of strength, notes Berger.

Using the material in certain objects, such as vehicles, could make them more energy efficient, and the foam’s regularly repeating cellular structure makes it simple to manufacture and scale to demand.
Sangaku (1847)
Japanese Temple Geometry problem on 3-4-5 Triangle posed by a 13-year old kid.

Find the radii of the circles.
old approach vs new approach

Old 3-4-5 △ vs New 1-2-3-4-5-6-7-8-9-10

why we should go with the new approach?

a) interesting
b) meaningful
c) broadening
d) challenging
a. 1… the radius of the incircle  \( \frac{a + b - c}{2} \)
b. 2… diameter of the incircle, the shortest length of the segment that divides the triangle into 2 equal areas*
c. 3… the shorter length of the triangle, but the number has the most interesting info  samples given
d. 4… the length of another leg, many interesting properties  homophone (death)  project like 3
e. 5… yes, it is the hypotenuse number, but it is also Euler line that goes through 1) orthocenter 2) circumcenter, 3) centroid 4) center of 9-point circle 5) Exeter point  project like 3
f. 6… the area of the triangle, and the total number of the internal circles that are tangent to two side (initially) then the rest of the circles are tangent to two sides and the previous circle  project like 3
g. 7… the sum of the two legs is obvious, but 7 is useful to neurologist to determine the onset of dementia (continue subtracting 7 from 100.) the dots on opposite sides of a die must sum to 7, there are many other items related to 7  project like 3
h. 8… dissection of a square with 8 acute triangles

8 is a lucky number in China  project like 3
i. 9… nine-point circle that can passes through nine significant concyclic points  project like 3
j. 10 = \( \frac{5!}{(3 \times 4)} \)  10 = \( 3 + \sqrt{4} + 5 \)  10 = \( 5!/4! \times \Gamma(3) \)
Why 3 is so interesting

1. 3 geometric problems of antiquity
   a. the trisection of the angle
   b. circle squaring
   c. Delian problem

2. Gauss proved that every integer is the sum of at most 3 triangular numbers.

3. Vinogradov Theorem states that all sufficiently large odd numbers are the sum of at most three primes

4. Fermat-Torricelli point of a triangle: the point such that the sum of its distances from the vertices of a triangle is a minimum.

5. Johnson’s theorem states that if three congruent circles intersect in a single point, the other three points of intersection lie on a circle congruent to the first three.

6. Challenge: using three 3’s and math symbols, find ways to represent 20.
   (there are more than $3 \times 3 + 3$ different ways)
In many languages, we have the positive, comparative and superlative. In English the sequence once-twice-thrice and goes no further. In Christianity, God is a trinity.
*In Greek mythology there were 3 Fates, 3 Furies, 3 Graces, 3 times 3 Muses, and Paris hard to choose between 3 goddesses. The natural world is 3 dimensional. The world is divided into three parts, the underworld, the earth, and the heaven. Dante’s masterpiece is divided into 3 segments: Inferno, Purgatorio, and Paradiso.
*There are the three “geometric-problems of antiquity and the trisection of the angle was one of them. Others are circle squaring and Delian problem. A circle can be drawn through any 3 points not on a straight line.
*There are just 3 tessellations of the plane with regular polygons, using equilateral triangles, squares, or hexagons as in a honeycomb.
*Gauss proved that every integers is the sum of at most 3 triangular numbers. 3 is the second triangular number.
A number is divisible by 3 if and only if the sum of its digits is divisible by 3. *3 is the first odd prime, the first Mersenne prime, & the first Fermat prime.
*Vinogradov Theorem states that all sufficiently large odd numbers are the sum of at most three primes.
It is the first member of an arithmetical progression of 3 primes, 3-5-7. The smallest magic square is of order 3.
“Three Legs of Man” has been the official symbol of the Isle of Man.
*Fermat-Torricelli point of a triangle: the point such that the sum of its distances from the vertices of a triangle is a minimum.
*Johnson’s Theorem states that if three congruent circles intersect in a single point, the other three points of intersection must lie on a circle congruent to the first three.
*Napoleon’s Theorem states that if three equilateral triangles of different sizes are joined point to point, their centers will form the vertices of the fourth equilateral triangle.
*Martin Gardner’s Three Prisoner Problem
*The most interesting challenge question: Using three 3’s and standard mathematical symbols, find 10 ways to represent 20.
Curry Triangle
AO =
GH =
DH =
AG =
Average of 3 segments?
\[
\frac{BG + CG + GH}{3} =
\]
Heronian Mean

$$H_M = \frac{1}{3} (A + \sqrt{AB} + B)$$

A square frustum, with volume equal to the height times the Heronian mean of the square areas.
Four Nurturing Geometry Problems

1. Given an equilateral triangle, the segments 3, 4, 5 are dropped from point P ⊥ to the sides, find the area of the triangle.
2. Given the semicircle, find the Geometric Mean, Arithmetic Mean, Harmonic Mean, & Root Mean Square.
3. Given a unit square, find the areas a, b, c, d.
4. Find the angle Z.
Conway Sequence

3
1 3
1 3 1 3
3 1 1 3
1 3 2 1 1 3
1 1 1 3 1 2 2 1 1 3
3 1 1 3 1 1 2 2 2 1 1 3
1 3 2 1 1 3 2 1 3 2 2 1 1 3
?

So what are the numbers for the next row?
Conway’s Game of Life

It is a cellular automaton and its evolution is determined by its initial state requiring no further input. Within the grid of square cells, they are either alive or dead. Every cell interacts with its eight neighbors, which are the cells that are horizontally, vertically, or diagonally adjacent based on the following rules:

1. Any live cell with fewer than two live neighbors dies, as if caused by underpopulation.
2. Any live cell with two or three live neighbors lives on to the next generation.
3. An live cell with more than three live neighbors dies, as if by overpopulation.
4. Any dead cell with exactly three live neighbors becomes a live cell, as if by reproduction.

The game made Conway instantly famous, but it also opened up a whole new field of mathematical research, the field of cellular automata ... Because of Life's analogies with the rise, fall and alterations of a society of living organisms, it belongs to a growing class of what are called "simulation games”

Ever since its publication, Conway's Game of Life has attracted much interest, because of the surprising ways in which the patterns can evolve. Life provides an example of emergence and self-organization. Scholars in various fields, such as computer science, physics, biology, biochemistry, economics, mathematics, philosophy, and generative sciences have made use of the way that complex patterns can emerge from the implementation of the game's simple rules.
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<td>74 75 76 77 78</td>
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Viviani Theorem & Ternary Plot

Viviani’s Theorem: The sum of distances of a point inside an equilateral triangle equals the length of its altitude.

flammability of

methane

It graphically depicts the ratios of the three variables as positions in an equilateral triangle. The point has a barycentric coordinates. It is used in physical chemistry, petrology, mineralogy, metallurgy, and other physical sciences to show the compositions of systems composed of three species. In population genetics, it is often called a de Finetti diagram. In game theory, it is often called a simplex plot.

Even in the field of medicine, when three different diagnostic methods are considered to determine the best possible solution for the patients, the barycentric plot is used as suggested treatments.

What do you call a triangle with only one angle 60°?
1) You have 100 lbs of potatoes, which are 99 percent water by weight. You let them dehydrate until they are 98 percent water. How much do they weigh now?

2) You drive from point A to point B at the rate of 60 mph and return the same route at the rate of 40 mph. What is the rate of the average mph for the entire trip?

3) Worshipful natives are rolling a giant statue across their island. They made an idol statue of you because they think you are a Math God. The statue rests on a slab, which rests on rollers that have a circumference of 1 meter each. How far forward will the statute have moved when the rollers have made 1 revolution? If you fail to get this EASY answer, the math-worshipping cannibals will be very upset and eat you alive.

4) How many times must an 8-toothed cogwheel rotate on its axis to circle around a 24-toothed cogwheel?

5) There are two concentric circles. The radius of the larger circle is R and the radius of the smaller circle is r. The length of the chord tangent to the inner circle is 20 inches. Find the area of the annulus.
# List of Notable Primes

1. Additive primes  
2. Annihilating primes  
3. Bell number primes  
4. Carol primes  
5. Centered decagonal primes  
6. Centered heptagonal primes  
7. Centered square primes  
8. Centered triangular primes  
9. Chen primes  
10. Circular primes  
11. Cousin primes  
12. Cuban primes  
13. Cullen primes  
14. Dihedral primes  
15. Double factorial primes  
16. Double Mersenne primes  
17. Eisenstein primes  
18. Emirps  
19. Euclid primes  
20. Even prime  
21. Factorial primes  
22. Fermat primes  
23. Fibonacci primes  
24. Fortunate primes  
25. Gaussian primes  
26. Generalized Fermat primes  
27. Genocchi number primes  
28. Gilda's primes  
29. Good primes  
30. Happy primes  
31. Harmonic primes  
32. Higgs primes for squares  
33. Highly cototent number primes  
34. Irregular primes  
35. (p, p−5) irregular primes  
36. (p, p−9) irregular primes  
37. Isolated primes  
38. Kynea primes

- 39. Left-truncatable primes  
- 40. Leyland primes  
- 41. Long primes  
- 42. Lucas primes  
- 43. Lucky primes  
- 44. Markov primes  
- 45. Mersenne primes  
- 46. Mersenne prime exponents  
- 47. Mills primes  
- 48. Minimal primes  
- 49. Motzkin primes  
- 51. Non-generous primes  
- 52. Odd primes  
- 53. Padovan primes  
- 54. Palindromic primes  
- 55. Palindromic wing primes  
- 56. Partition primes  
- 57. Pell primes  
- 58. Permutable primes  
- 59. Perrin primes  
- 60. Pierpont primes  
- 61. Pillai primes  
- 62. Primes of the form \( n^4 + 1 \)  
- 63. Primeval primes  
- 64. Primorial primes  
- 65. Proth primes  
- 66. Pythagorean primes  
- 67. Prime quadruplets  
- 68. Primes of binary quadratic form  
- 69. Quartan primes  
- 70. Ramanujan primes  
- 71. Regular primes  
- 72. Repunit primes  
- 73. Primes in residue classes  
- 74. Right-truncatable primes  
- 75. Safe primes  
- 76. Self primes in base 10  

77. Sexy primes  
78. Smarandache–Wellin prime  
79. Solinas primes  
80. Sophie Germain primes  
81. Star primes  
82. Stern primes  
83. Super-primes  
84. Supersingular primes  
85. Swinging primes  
86. Thabit number primes  
87. Prime triplets  
88. Twin primes  
89. Two-sided primes  
90. Ulam number primes  
91. Unique primes  
92. Wagstaff primes  
93. Wall-Sun-Sun primes  
94. Wedderburn-number primes  
95. Weakly prime numbers  
96. Wieferich primes  
97. Wilson primes  
98. Wolstenholme primes  
99. Woodall primes
So many primes, but so little time!
Let's look at some primes just for fun.

Emirps: 13, 17, 31, 37, 71, 73, 79...

Do you know your twin primes, cousin primes, & sexy primes? or how about Swinging Primes? primes which are within 1 of a swinging factorial: n! ± 1
2, 3, 5, 7, 19, 31, 71...

Happy Primes: numbers that are both happy and prime.
7, 13, 19, 23, 31, 78, 103...

379009 what do you think of this prime number?

867-5309 Jenny's phone number 5309 & 8675309 are both prime numbers

613 producing different classes of numbers by rearranging its digits.
136 (triangular number) & the square of a number 361 = 19^2

Mersenne Primes... most famous and important primes

\[ M_p = 2^p - 1 \] (GIMPS) 3, 7, 31, 127...

Curtis Cooper Prime number hunter
Why prime numbers still fascinate mathematicians, 2,300 years later

Robert Langlands received the Abel Prize this year. Langlands' research demonstrated how concepts from geometry, algebra and analysis could be brought together by a common link to prime numbers.

Euclid proved the infinitude of primes but history suggests it was Eratosthenes who gave us the sieve to quickly list the primes.

Gauss worked on large prime numbers by counting them, one chiliad (a group of 1000) at a time. He counted the primes up to 1,000, then the primes between 1,000 and 2,000, then between 2,000 and 3,000 and so on.

A century after Gauss' first explorations, his law was proved in the "prime number theorem." The percent error approaches zero at bigger and bigger ranges of primes. The Riemann hypothesis, a million-dollar prize problem today, also describes how accurate Gauss' estimate really is.

Today, our data sets come from computer programs rather than hand-cut stencils, but mathematicians are still finding new patterns in primes. Except for 2 and 5, all prime numbers end in the digit 1, 3, 7 or 9.

Yitang Zhang...distribution of primes numbers describing the gaps between prime numbers making a step forward in understanding the twin primes conjecture and dealing with separation of extremely large prime numbers.

Solomon Wolf Golomb received his PhD in mathematics from Harvard with a dissertation on "Problems in the Distribution of the Prime Numbers", best known for his works on mathematical games. Most notably, he invented Cheskers in 1948. He also fully described pentominoes in 1953. He specialized in problems of combinatorial analysis, number theory, coding theory, and communications. His game of pentomino inspired Tetris. Golomb was the inventor of Golomb coding, a form of entropy encoding. Golomb rulers, used in astronomy and in data encryption,
Grandi’s Series

What is the sum of the following:

1 - 1 + 1 - 1 + 1 - 1 + ... = ?

The answer could be

0, 1, 1/2, 2/3

or something else?

1/2 is called Cesaro sum
Euler and Ramanujan for 1/2
Bernoulli was for 2/3
Grandi’s Series

1 - 1 + 1 - 1 + 1 - 1...

This series is very controversial. Here are reasons why.

case 1

\[(1-1)+(1-1)+(1-1)\ldots = 0\]

case 2

\[1-(1-1)-(1-1)-(1-1)\ldots = 1\]

case 3 (Cesaro sum) = \(\frac{1}{2}\)

Euler, Ramanujan, and other famous mathematicians agree with Cesaro method, but Euler came up with the following:

\[\frac{x}{1+x} = x - x^2 + x^3 - x^4 + x^5\ldots\]

1 + x let \(x = 1\) then 1 - 1 + 1 - 1 - 1...

so \(\frac{x}{1+x} = \frac{1}{1} = \frac{1}{2}\)

\[1 + x \quad 1 + 1\]

Case 4  Bernoulli’s idea

\[\frac{(1 + x)}{(1 + x + x^2)} = \frac{2}{3}\]
Cycloid – The Helen of Geometers

Galileo - originator of the term
The Area under the Curve = $3\pi r^2$
Arc Length of the Curve = $8r$
Tautochrone curve = the curve for which the time taken by an object sliding without friction in uniform gravity to its lowest point regardless of its starting point.

Brachistochrone curve = curve of fastest descent, is the one lying on plane between a point A and a lower point B, where B is not directly below A, on which a bead slides frictionlessly under the influence of a uniform gravitational field to a given end point in the shortest time.

Cycloid = the curve traced by a point on the rim of a circular wheel as the wheel rolls along a straight line without slipping.
According to Schlafli symbols, what do they represent?

1. \{3\}
2. \{4\}
3. \{4, 3\}
4. \{17\}
5. \{5/2\}
6. \{3, 3\}
Schläfli symbol

It is a notation of the form \( \{p,q,r,\ldots\} \) that defines regular polytopes and tessellations, starting with \( \{p\} \) for a p-sided regular polygon that is convex. For example, \( \{3\} \) is an equilateral triangle, \( \{4\} \) is a square, \( \{5\} \) a convex regular pentagon and so on. So the famous Gauss’ heptadecagon is \( \{17\} \). Regular star polygons are not convex, and their Schläfli symbols \( \{p/q\} \) contain irreducible fractions \( p/q \), where \( p \) is the number of vertices. For example, \( \{5/2\} \) is a pentagram.

A regular polyhedron that has \( q \) regular \( p \)-sided polygon faces around each vertex is represented by \( \{p,q\} \). For example, the cube has 3 squares around each vertex and is represented by \( \{4,3\} \). \( \{3,3\} \) is a tetrahedron.
Octal to binary conversion
To convert octal to binary, replace each octal digit by its binary representation.
Example: Convert 518 to binary: \(5_8 = 101_2\) \(1_8 = 001_2\) Therefore, \(51_8 = 101 001_2\).

Binary to octal conversion
The binary digits are grouped by threes, from right to left. Then replace each trio with the equivalent octal digit. For instance, convert binary 1010111100 to octal:
\[
\begin{array}{ccccccccc}
0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0
\end{array}
\]
Therefore, \(1010111100_2 = 1274_8\).

Binary to Hexadecimal
Example 1. Consider Binary: 1000100100110111
STEP 1 Break the Byte into 'quartets' - \(1000\ 1001\ 0011\ 0111\)
Hex F E D C B A 9 8 7 6 5 4 3 2 1
Binary 1111 1110 1101 1100 1011 1010 1001 1000 0111 0110 0101 0100 0011 0010 0001
Dec 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
STEP 2 Use the table above to covert each quartet to its Hex equivalent - 8937
Therefore ... \(1000\ 1001\ 0011\ 0111 = 8937_{Hex}\)

Hexadecimal to binary
There is no easy way to remember the Hex to Binary conversions for A to F. You need to learn them so you can automatically write them down without thinking. Once you have learnt the A to F conversion the process of general conversion from Hex to Binary and back becomes very simple. (So learn them!) You can surely remember \(A = 1010\)

Binary to decimal
\[
\begin{array}{cccccccc}
128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\
\downarrow & & & & & & & \\
1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
\end{array}
\]
\[
128 + 16 + 8 + 2 + 1 = 155 \text{ in base 10.}
\]
Another method is to use Synthetic Division. Divide the binary number by 2.
\[
2 \mid 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \\
\downarrow \ 2 \ 4 \ 8 \ 18 \ 38 \ 76 \ 154 \\
1 \ 2 \ 4 \ 9 \ 19 \ 38 \ 77 \ 155 \leftarrow
\]

Decimal to binary: Divide the numbers repeatedly by 2 and the remainders in reverse is the binary number.
Number System Conversions

Fill in the blanks to each of the statements, converting the values from one number system to another.

(1) 10 in binary is equal to ____ in octal and ____ in hexadecimal
(2) 11 in binary is equal to ____ in hexadecimal and ____ in decimal
(3) 10 in octal is equal to ____ in decimal
(4) 1010 in binary is equal to ____ in hexadecimal
(5) 71 in octal is equal to ____ in binary and ____ in decimal
(6) 132 in octal is equal to _____ in binary
(7) 170 in octal is equal to ____ in hexadecimal
(8) 100110010 in binary is equal to ____ in octal and ____ in decimal
(9) 17A in hexadecimal is equal to ____ in decimal and ____ in octal
(10) 447 in decimal is equal to ____ in octal and ____ in binary
(11) 9 in decimal is equal to ____ in binary and ____ in octal
(12) 13 in octal is equal to ____ in binary
(13) 100011 in binary is equal to ____ in octal
(14) 1011100 in binary is equal to ____ in decimal
(15) 10110011 in binary is equal to ____ in decimal and ____ in octal
(16) 278 in decimal is equal to ____ in binary and ____ in hexadecimal
(17) 451 in decimal is equal to ____ in hexadecimal and _____ in binary
(18) 2E2 in hexadecimal is equal to ____ in binary and ____ in decimal
(19) 32D in hexadecimal is equal to ____ in octal and ____ in binary
Coffin (Killer) Problems

1) Which is larger $\log_2 3$ or $\log_3 5$?  Hint: $3/2$

2) How many digits does the number $125^{100}$ have?  Hint: $2^{10} = 1024$

3) Solve the equation:

$$x^n = x \quad \text{for} \quad x = ?$$
An infinite tower of powers of $x$

\[ \ldots \]

\[ x \]

\[ x \]

\[ x \]

\[ X = A \]

Find the value of $x$ so that this tower is equal to 2.

\[ x^A = A \quad \text{then} \quad x = A^{1/A} \]

If $A = 2$, then $x = \sqrt{2}$

if $A = 4$ then $x = 4^{1/4} = \sqrt{2}$

Does this mean there are two solutions for this problem?
The next number?

1, 2, 4, 8, 16, ?

Moser’s circle problem: determine the number of pieces into which a circle is divided if \( N \) points on its circumference are joined by chords with no three internally concurrent.

\[
g(n) = nC_4 + nC_2 + 1
\]
1. Binomial Coefficients \((a+b)^n\)
   Use Newton’s Expansion Method when \(n=\text{fraction}\)
2. Sum of Coefficients \(2^n\)
3. Vedic Math \(11^n\)
4. Fibonacci Sequence...Elliott Wave
5. Triangular Numbers...handshake
   \[n(n+1)/2 = \text{Gaussian sum}\]
   \[\text{Num} = \Delta + \Delta + \Delta\]
6. Moser’s Circle Problems
7. Manhattan Distance \((x+y)! / (x!y!))\)
8. Probability of Heads and Tails
1, 2, 4, 8, 16, __
the number of divisors based on factorials sequence

1! 2! 3! 4! 5! 6!
1 2 4 8 16 30
3! = 3 x 2 x 1 = 6
numbers of divisors = 4
6, 3, 2, 1
4! = 4 x 3 x 2 x 1 = 24
numbers of divisors = 8
24, 12, 8, 6, 4, 3, 2, 1
5! = 5 x 4 x 3 x 2 x 1 = 120
number of divisors = 16
6! = 6 x 5 x 4 x 3 x 2 x 1 = 720
number of divisors = 30
Who is Charles Dodgson?

1. mathematician
2. creator of the doublet
3. author of Alice in Wonderland
4. his pen name is Lewis Carroll
5. worked on Cryptography & Nyctograph
   rule: change one letter at a time (no proper noun)
   HEAD - heal - teal - tell - tall - TAIL
   ideal doublets... n letters with n links
   (shorter the better)

Later development:
1. you may rearrange the letters of the word or change one letter of the word. You can not do both at the same time.
2. You may add the letter.
3. You may subtract the letter.
4. You may alternate from synonym to antonym
5. unintended contributions to the information theory: Lee distance, Hamming distance, Levenshtein distance, coding and decoding,...

COLD ______ ______ ______ WARM

Donald Knuth (famous mathematician and computer scientist) states that there is one word that will not work as a doublet starting or ending word: ALOOF
Change GRASS to GREEN!

GRASS  GRASS  GRASS  GRASS  GRASS  GRASS
↑
CRASS  GROSS  GLASS  BRASS  GRABS  CRASS
↑
CREASE  GROWS  GLOSS  BRANS  CRABS  CRAPS
↑
TRESS  GROWN  FLOSS  BRAND  CRIBS  TRAPS
↑
TREES  GROIN  FLOES  BLAND  CRIES  TRIPS
↑
FREES  GROAN  FLEES  BLEND  CRIER  TRIES
↑
FRED  GROAT  FREES  BLEED  CRIED  TREES
↑
GREED  GREAT  FREED  BREED  CREED  TREED
↑
GREEN  GREET  GREED  GREED  GREED  GREED
↑
GREEN  GREEN  GREEN  GREEN  GREEN  GREEN
THE BEST DOUBLET
requires a higher level of vocabulary

GRASS

TRASS volcanic tuff used in making water-resistant cement & mortar

TRESS long locks or curls of hair.

TREES

TREEN small woodenware

GREEN
New Development from Doublets

Hamming Distance... equal length strings

The Hamming distance between:

* 1011101 and 1001001 is 2.
* 2173896 and 2233796 is 3.
* "toned" and "roses" is 3.

Lee Distance... used in coding

the Lee distance between

* 3340 and 2543 is 6.
  \[1+2+0+3 = 6.]\n* abcd and cdgc is 7
  \[2 + 2 + 0 + 3 = 7\]

Levenshtein Distance... spell checkers

The edit distance between two strings is given by the number of operations needed to transform one string into the other, where an operation is an insertion, deletion, or substitution of a single character at a time. It is often used in applications that need to determine how similar, or different, two strings are.
Steganography is the art and science of writing hidden messages in such a way that no one, apart from the sender and intended recipient, suspects the existence of the message, a form of security through obscurity. The word steganography is of Greek origin and means "concealed writing" from the Greek words steganos meaning "covered or protected", and graphei meaning "writing". The first recorded use of the term was in 1499 by Johannes Trithemius in his Steganographia, a treatise on cryptography and steganography disguised as a book on magic. It was the text about the art of hiding messages in text. Generally, messages will appear to be something else: images, articles, shopping lists, or some other covert text and, classically, the hidden message may be in invisible ink between the visible lines of a private letter.
Codes & Secret Writings

1. Caesar Cipher

2. Rail Fence

3. Pigpen

4. Polybius

5. Lewis Carroll’s Vigenere & Nyctograph
Coding and Decoding questions can be divided into following types:

1. Letters
2. Numbers
3. Symbols
4. Group of Words
5. Mathematical

Sample Question

In a certain language, TEARS is coded as 18, so how will WATER be coded?
Message from the Zodiac Killer
Interactions of Art, Science & Mathematics

Visual Phenomena & Optical illusions
Michael Bach

1. Different movements
2. Biological motion
3. Checker Shadow (MIT)
4. Curry Triangle
5. Necker Cube & Missing Corner
6. Rotating Silhouette
7. Anamorphosis
8. Jastrow Illusion
9. TSP AxiDraw® example - drawing the Mona Lisa using TSP paths
10. Trompe-l’œil How to Draw 3D Letter M - Drawing with pencil - By Vamos
11. Sugihara’s Ambiguous Cylinders
12. Fantastic spider weaves a web
13. Eshima Ohashi Bridge
1. Blivet
2. Jastrow Illusion (psychologist)
3. Penrose triangle (Mathematical physicist)
4. Ebbinghaus illusion (psychologist)
5. Kanizasa triangle (psychologist)
6. Curry Triangle (neuropsychiatrist)
7. Sander Illusion (psychologist)
8. Poggendorff Illusion (physicist)

Blivet
1. something annoying, ridiculous, or useless.
2. something for which one cannot find a word;
3. an unpleasant or unsolvable situation or problem.
just a martini glass
Trompe L’oeil

an art technique that uses realistic imagery to create the optical illusion.
Trompe L’oeil

lesmaterialistes.com
Look at the chart and say the **COLOUR** not the word

YELLOW  BLUE  ORANGE  BLACK  RED  GREEN  PURPLE  YELLOW  RED  ORANGE  GREEN  BLACK  BLUE  RED  PURPLE  GREEN  BLUE  ORANGE

Left - Right Conflict
Your right brain tries to say the colour but your left brain insists on reading the word.
Eshima Ohashi Bridge in Japan
“A calm and modest life brings more happiness than the pursuit of success combined with constant restlessness.” A handwritten note on happiness that Albert Einstein gave to a bellboy in Japan in 1922 sold for $1.56 million at an auction in Jerusalem.
<table>
<thead>
<tr>
<th>Mathematician</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>PYTHAGORAS</td>
<td>(570 BC-495 BC)</td>
</tr>
<tr>
<td>EUCLID</td>
<td>(325 BC-265 BC)</td>
</tr>
<tr>
<td>LEONARDO PISANO</td>
<td>(1175-1250)</td>
</tr>
<tr>
<td>BIGOLLO</td>
<td></td>
</tr>
<tr>
<td>RENÉ DESCARTES</td>
<td>(1596-1650)</td>
</tr>
<tr>
<td>ISAAC NEWTON</td>
<td>(1642-1727)</td>
</tr>
<tr>
<td>GOTTFRIED WILHELM LEIBNIZ</td>
<td>(1646-1716)</td>
</tr>
<tr>
<td>LEONHARD EULER</td>
<td>(1707-1783)</td>
</tr>
<tr>
<td>CARL FRIEDRICH GAUSS</td>
<td>(1777-1855)</td>
</tr>
<tr>
<td>ADA LOVELACE</td>
<td>(1815-1852)</td>
</tr>
<tr>
<td>G. F. BERNHARD RIEMANN</td>
<td>(1826-1866)</td>
</tr>
<tr>
<td>AMALIE EMMY NOETHER</td>
<td>(1882-1935)</td>
</tr>
<tr>
<td>ALAN TURING</td>
<td>(1912-1954)</td>
</tr>
</tbody>
</table>

**Fermat’s Last Theorem**

There are no three positive integers $x$, $y$, and $z$ for which

$$x^n + y^n = z^n$$

for any integer $n > 2$
They say "music is mathematics for the soul: Mathematics is music for the mind". And we know math and music can illuminate and enrich our lives.

Years may wrinkle the skin, but to give up or lose enthusiasm for life, music, and math really wrinkles our soul.

We believe "the empire of the future is the empire of the knowledge, imagination and creativity."

We stand on the shoulder of history to face the destiny of our lives. We must meet the challenges of the day with determination. So let us aspire to inspire others before we expire.

Age infans meum ignem incende. This may sound Greek to some of you, but this is one of the most famous and inspiring Latin phrases. It simply says "Come on, baby. Light my fire!"
Chris Michaels' “TV control device” (patented in 1976)

Do you remember 8 track cassette tape?

In an address to the Mathematical Association of England on the importance of education in 1917, Alfred North Whitehead argued that "the basis of invention is science, and science is almost wholly the outgrowth of pleasurable intellectual curiosity."
All you need is

\[ y = \frac{1}{x} \]

\[ x^2 + y^2 = 9 \]

\[ y = | -2x | \]

\[ x = -3| \sin y | \]
Mnemonic Device for e

2 term serving
7th President Andrew Jackson
1828 first elected
1828 elected twice so repeat
45-90-45 isosceles right triangle
235 Fibonacci numbers without 1
360 perigon
28 one year before stock market crash
747 Jumbo Jet
1352 the end of the Bubonic Plague in Europe
or Fibonacci number 2 moved next to 5
662 take away death (4) from the Devil 666
or border line FICO credit score
4977 \(49 = 7 \times 7\)
5 pentagram (protection against evil)
secret symbol for Pythagoreans
724 link to Hell (xkcd)
709 inheritance transfer tax form
36 interior angle for regular pentagram
999 Britain’s emergency telephone number
or Herman Caine’s political slogan
No! minus death

Protection against Evil
the telephone area code in the Canadian province of Newfoundland or Inheritance tax form

interior angle of a point

Nine Hours, Nine Persons, Nine Doors
Nintendo's graphic adventure game

British Emergency Phone Number
Fill in the blanks with appropriate words that will connect adjoining words.

Bull (    ) House (    ) Yard (    ) Bath
(    ) Fall (    ) Side (    ) Way (    )
Down (    ) Pants (    ) Case (    )
Door (    ) hill*(    ) Out (    ) Cat (    )
Waiting (    ) Service (    ) Girl (    )
Master (    ) Board (    ) On (    )
Kick (    ) Firm (    ) House (    )

Pooper

Math Question:

Find the number of distinguishable permutations of the letters in the word “pooper”.