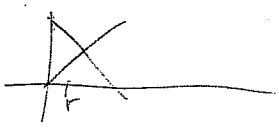
- Hand out figure, do demos.
- Ask: "What do you think?"
  "What are some ways to attack the problem?

Algebraically



(1) Eq. of line $\left[y = \frac{-1}{r}x + 1\right]$

(2) Intersection $\left(\frac{r}{r+1}, \frac{r}{r+1}\right)$

(3) Reflect $\left(\frac{1}{r+1}, \frac{1}{r+1}\right)$

(4) Eq. of line $\left[y = -rx + 1\right]$

(5) Intersection $\left[x = \frac{1}{r}\right]$.

With

Projective Geometry

① One way: All normal points + lines, but parallel lines will intersect. This is because there are some more points - "at ∞."

② (Look out window)

③ Projections are important

Define $f_p : \ell \longrightarrow \ell'$

④ How does this picture look familiar?

- What are the lines?
- What is the point?

We will use this!

Definition [Harmonic Pairs]

$\{-1, 1\}$ and $\{r, \frac{1}{r}\}$ are a harmonic pair.

Two pairs of points are harmonic if, when rescaled, they look like $\{-1, 1\}$, $\{r, \frac{1}{r}\}$.

Some harmonic pairs —

$\{-1, 1\}$ and $\frac{1}{2}, 2$

$\frac{1}{3}, 3$

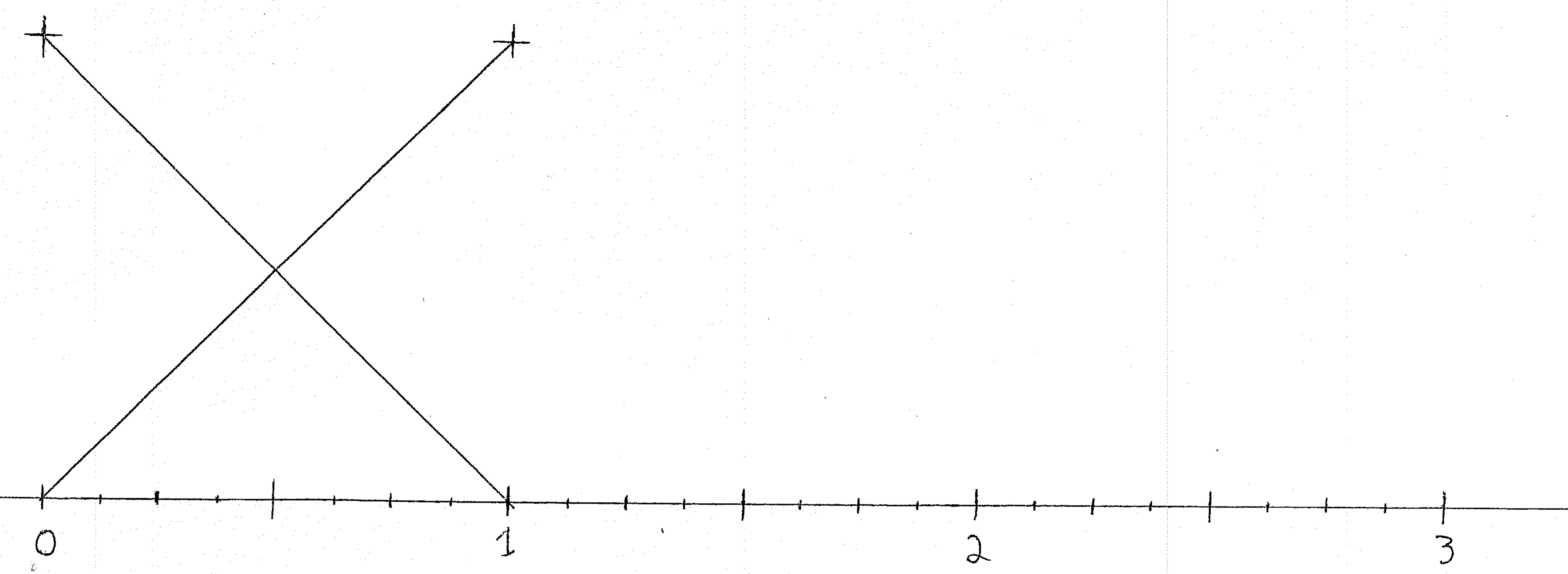
$0, \infty$.

Key result :

Projections send harmonic pairs to harmonic pairs.

Try it out!!

This explains it.


0
1
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