# Mathematical Auction

Math Circle - Summer 2009

## Rules:

(i) We divide into teams and work for a fixed amount of time to solve the problems below.

(ii) Each time is given starting money of $1000.

(iii) The best solution to a problem is worth $200.

(iv) The problems are put up for auction in the order given. The team with the highest bid is allowed to present their solution.

(v) The problem is then put up for bid again (and again) but this time the solution must be better than the previous solution.

(vi) When no other team wants to buy the problem, the team with the best solution collects the value of the problem. Every team that "bought" the problem pays for their bid, even if they were not the winning solution.

(vii) If a team can show that they have the best solution (by showing that their solution can not ever be improved) then that team is eligible for $50 prize money for the problem.

## Problems

1. Divide a square into the least number of acute triangles (an acute triangle is one with all acute angles).

2. Using only the digits 1,9,8 and 4, in this given order, and the four arithmetic operations ($+$, $-$, $\times$, $\div$), write as many consecutive natural numbers as possible starting with 1.

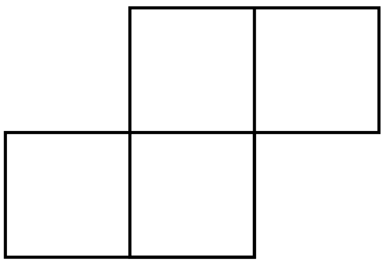
   For example, $(1 + 9)/(8/4) = 5$.

3. Draw 7 lines in the plane such that you produce as many triangles as possible.

4. Find as many solutions to the equation $x^2 + y^2 = z^2$, with $x, y, z$ natural numbers, with no number greater than 50.

5. Two circles with different radii are centered at the same point. Pick four distinct points on the outer circle and two distinct points on the inner circle. Connect all the points to get lines. What is the minimal number of straight lines?

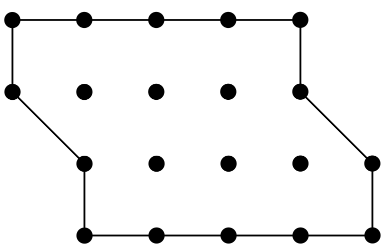
6. Find the maximum number of the figure below that can be placed, without overlapping, inside a $10 \times 10$ table.



7. Start with the number 1234512345123451234512345 and cross out ten digits so that the remaining number is as large as possible.

8. Let $n = ABC$ be a three-digit number, where $A$, $B$ and $C$ are the three digits. Compute the largest possible value of:
$$\frac{n}{A+B+C}$$

9. The famous chef, Patty Cake, cooks a cake that has the shape below. This cake is to be cut into four equal parts of exactly the same size and shape. Find as many different ways to cut this cake into four pieces of the same size and shape.



10. TOURNAMENT

    The rules are as follows:

    (a) Every two teams play exactly two games, one game as visiting team and the other as home team;

    (b) The team does not travel if it plays as home team during the week (seven days from Sunday to Saturday) and it can play multiple games as the home team

    (c) A team can play as the visiting team in multiple, up to seven games within one week.

    What is the maximal number of teams you can fit in 4 weeks?

11. Draw a triangle with vertices $ABC$ and pick a point $D$ different from $B$ and $C$ on side $BC$. Connect $AD$ using a segment. Then pick three points $EFG$ on segment $AD$ different from $A$ and $D$ and connect $EFG$ with $B$ and $C$ separately.

    Find the most triangles as possible in such a graph.