Russian Circle
Home Problems

Math Circle - Summer 2009

Instructions: Work as many problems as you can. Some of these problems are quite difficult. Even if you can not completely solve a problem, try to learn as much as you can about the problem.

1. Find all five digit numbers which, when divided by 4, yield a 5-digit number that is the same as the original, but with the digits reversed.

2. Two large jars contain 1 liter of water each. Half of the water from the first jar is poured into the second one; then one third of the water from the second jar is poured into the first one; then one fourth of the water from the first jar is poured into the second one; and so on. How much water is contained in each jar after repeating this operation 100 times?

3. The town of Smallville has exactly 15 telephones. Can these telephones be connected by wires so that each telephone is connected with exactly five others?

4. The side AC of triangle ABC has length 3.8, and side AB has length 0.6. If the length of BC is an integer, what is this length?

5. Find all natural numbers n such that $2^n > 2n + 1$. Prove that your answer is correct.

6. Below is an addition table.

+	E	7	G	H
A	7			15
3			11	
C				
6				16

The sum of the numbers in the bold square equals 200.

Determine, if possible, the values of all letters and all missing elements in the table.

7. A farm keeps some chickens. The chicken food will last 20 days after 75 chickens are sold or last 15 days after 100 new chickens are brought to the farm. How many chickens are there on the farm right now given all chickens have the same amount of food every day.

8. Given that $\frac{a^3+b^3+c^3-3abc}{a+b+c} = 3$ what is $(a-b)^2 + (b-c)^2 + (a-b)(b-c)$?

9. RED BALLS AND BLACK BALLS

   There are r red balls and b black balls in the bag. Randomly take out n balls $(n \leq r+b)$. What is the probability that you pick exactly k $(k \leq r)$ red balls out of n balls?

   Give two answers. In the first case, each time you draw a ball, record what color the ball is and then put the ball back.

   In the second case, each time you draw a ball, keep the drawn balls in a pile and do not replace the ball into the bag.

10. Sandy tosses a fair coin $n+1$ times and counts the number of heads she gets. Sue tosses the coin n times and counts the number of heads she gets. What is the probability that Sandy get more heads than Sue does?