Mathematical Battle

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Rules:

- You will be divided into two teams. You will be given approximately one and half hours to work on the problems as a team.

- There will be two judges who decide on scores. The decisions of the judges are final.

- Each team selects a captain who serves as spokesperson for the team and also participates in the captain's contest.

- The battle begins with a short question to be answered on the spot. The victor decides whether to begin with the right to challenge or to pass this right to the other team.

- At each stage of the battle, the team with the right to challenge chooses a problem from those that have yet to be presented and challenges the opposing team to present a solution.

- When challenged, the opposing team may choose to accept the challenge, in which case they present a solution. They may also opt to return the challenge, in which case the original team must attempt to present a solution.

- The team presenting a solution nominates one member who has not yet discussed a problem to provide an explanation. This person has up to five minutes to present as complete a solution to the problem as they are able. Drawing and writing equations is included in this five minutes. The presenter may briefly discuss the problem with their team before stepping to the board, but they may not consult with their team while describing their solution.

- The other team then nominates one member who has not yet discussed a problem to respond to the solution just presented. This person has up to three minutes to point

out any flaws or omissions or even supply an alternate solution. The respondent can discuss their rebuttal briefly with their team but may not consult their team while speaking.

- After the presentation and rebuttal, the judges may pose questions to one or both of the speakers.

- The judges then award the points available among the three parties involved: the presenting team, the rebutting team and the judges. Each problem is initially worth 7 points. However, should a team return a challenge, the value of the problem increases to 10 points.

- With one exception, the right to challenge then passes to the next team. Exception: when a team returns a challenge and the original team is unable to make significant progress toward a solution (defined as receiving 3 or fewer points), the right to challenge remains with the original team.

Problems:

1. You are given two glass jugs. Each contains the same volume, V, of liquid. One jug contains pure alcohol, and the other jug contains pure water. A modest quantity, Q, of water is poured from the water jug into the alcohol jug, which is then thoroughly mixed. The same modest quantity, Q, of (now diluted) alcohol is then poured back into the water jug to equalize the volumes of the jugs at their initial levels. Which one of the following is true and why?

   (A) The concentration of alcohol in the alcohol jug is **higher** than the concentration of water in the water jug.

   (B) The concentration of alcohol in the alcohol jug is **lower** than the concentration of water in the water jug.

   (C) The concentration of alcohol in the alcohol jug is **the same** as the concentration of water in the water jug.

2. What is the sum of integers from 1 to 100?

3. Picture a $10 \times 10 \times 10$ "macro-cube" floating in mid-air. The macro-cube is composed of $1 \times 1 \times 1$ "micro-cubes", all glued together. Weather damage causes the exposed (outermost) layer of micro-cubes to become loose. This outermost layer falls to the ground. How many micro-cubes are on the ground?

4. How many degrees (if any) are there in the angle between the hour and minute hands of a clock when the time is a quarter past three?

5. Inside of a dark closet are five hats: three blue and two red. Three smart men go into the closet, and each selects a hat in the dark and places it unseen upon his head. Each man knows both that the closet contains three blue hats and two red and that the other two have the same knowledge.

   Once outside the closet, no man can see his own hat. The first man looks at the other two, thinks, and says, "I can't tell what color my hat is." The second man hears this, looks at the other two, and says, "I cannot tell what color my hat is either." The third man is blind. The blind man says, "Well, I know what color my hat is." What color is his hat, and how does he know?

6. You have a string-like fuse that burns in exactly one minute. The fuse is inhomogeneous, and it may burn slowly at first, then quickly, then slowly, and so on. You have a match, and no watch. How do you measure exactly 30 seconds?

7. You hire a man to work in your yard for 7 days. You wish to pay him in gold. You have one gold bar with seven parts - like a chocolate bar. You wish to pay him one gold part per day, but you may snap the bar in only **two places**. Where do you snap the bar so that you may pay him at the end of each day, and so that on successive days he may use what you paid him previously to make change?

8. Can the mean of any two consecutive prime numbers ever be prime? Why?

9. You start with a single lily pad sitting on an otherwise empty pond. You are told that the surface area of the lily pad doubles every day and that it will take 30 days for the single lily pad to cover the surface of the pond.

   If instead of one lily pad you start with eight lily pads (each identical in characteristics to the original single lily pad), how many days will it take for the surface of the pond to become covered?

10. A nail is climbing up a 11-foot pole. It climbs up by three feet every day. Each night it sleeps. While sleeping, it slides down by one foot. When does it reach the top of the pole?