Basic Questions About the Universe

- What is the shape of the Earth?
- What is size of the Earth?
- How far is it from the Earth to the Moon?
- How far is it from the Earth to the Sun?
- What is the speed of light?
- How far is it from the Sun to other stars?

Astrometry is the study of these quantities. Nowadays these quantities are known.

- How were they calculated?
- This quantities are far too large to measure directly.
- Using some math we can calculate the quantities indirectly.

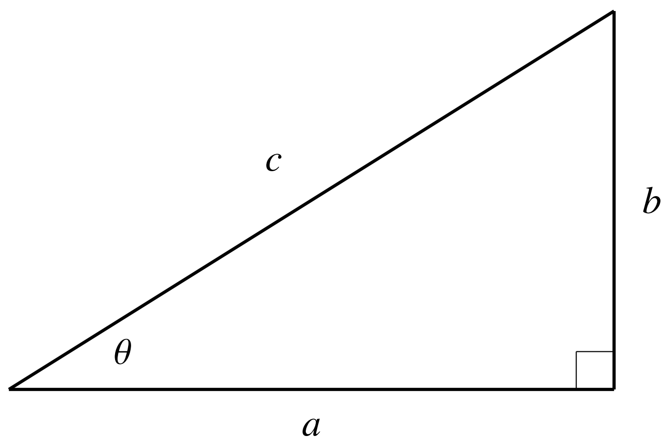
First step: the Earth

- **Aristotle** (384-322 BCE) gave a simple argument demonstrating why the Earth is a sphere.

Aristotle reasoned that lunar eclipses were caused by the Earth's shadow on the moon. He then observed that the shadow of the earth on the moon was always a circular arc. This meant that the Earth was most likely a sphere.

An important tool: Trigonometry

- **Trigonometry** is the study of the relationship between the sides and angles of a triangle.
- The basic trig functions, Sine, Cosine, and Tangent relate the acute angle of a right triangle to the ratio of its sides.



a = Adjacent

b = Opposite

c = Hypotenuse

$$\sin\theta = \frac{b}{c} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{a}{c} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{b}{a} = \frac{\text{Opposite}}{\text{Adjacent}}$$

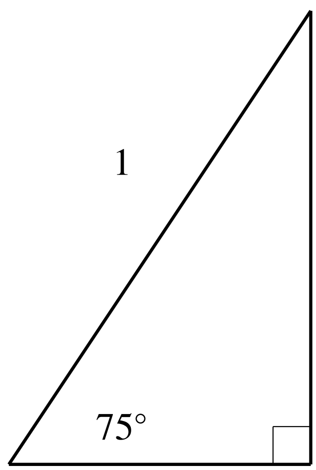
Mnemonic device : SOHCAHTOA

Calculate the following (make sure the calculator is in degrees)

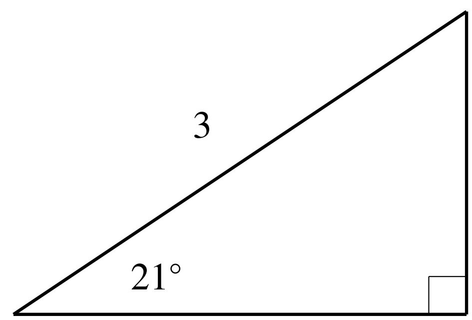
- $\sin 72^\circ =$______________
- $\cos 25^\circ =$______________
- $\tan 45^\circ =$______________

Trigonometry

Find x

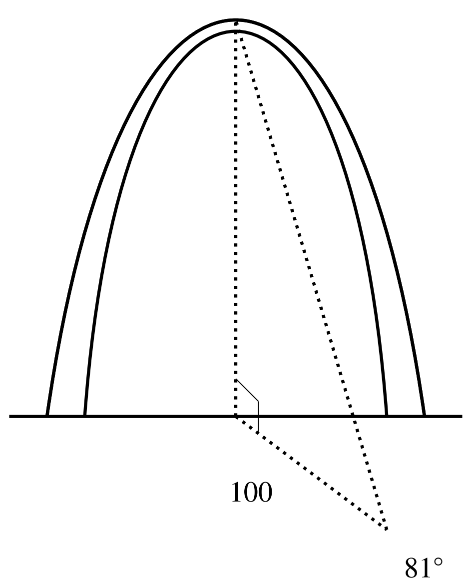


$x =$ ________



$x =$ ________

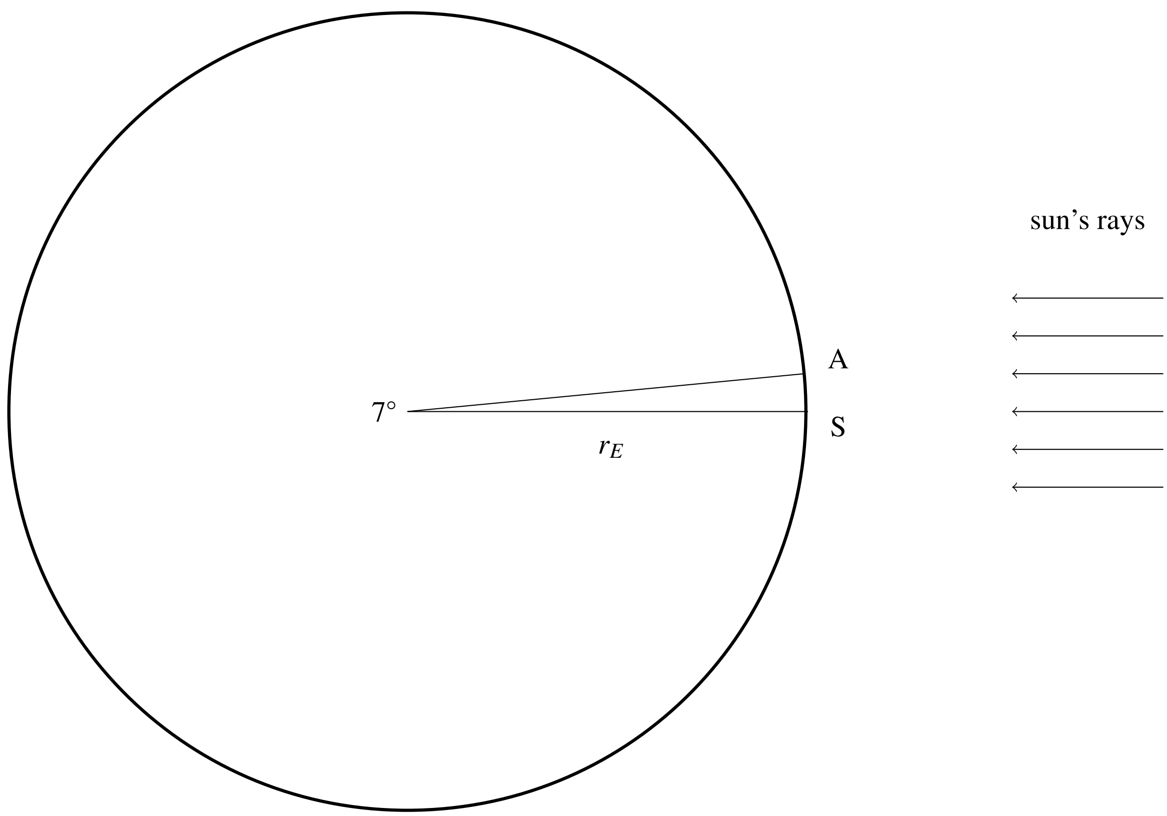
Question: Standing 100 ft away from the center of the base of the Saint Louis Arch, you measure an angle of 81° to the top. What is the height of the Arch?



Radius of the Earth

Eratosthenes (276-194 BCE) calculated the Earth's radius.

- Eratosthenes knew of a well in Syene that at noon on the summer solstice (June 21) reflected the suns light.
- He observed that a well in Alexandria on June 21 did not reflect the sun at noon.
- Using the shadow, he measured that the sun was at an angle of about 7° from the vertical on June 21 in Alexandria.
- He then measured the distance from Alexandria to Syene to be 5000 stadia (about 460 miles).

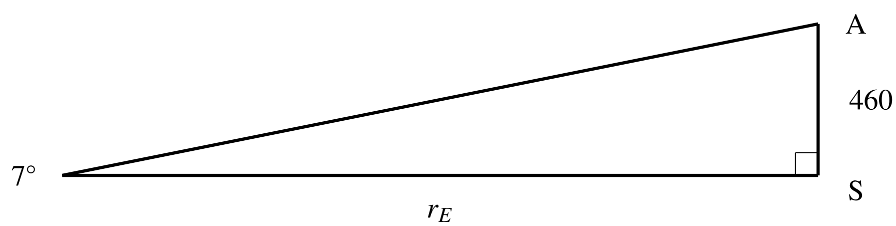


The sun's rays hit Syene (S) vertically, but Alexandria (A) at 7°.

Radius of the Earth

Eratosthenes measurements:

- Distance from A to S = 460 miles
- Angle at the center = 7°



- Calculate the radius of the Earth,

$$r_E = \underline{\hspace{4cm}}.$$

- Today the radius of the Earth calibrated with great precision using satellites is around

$$\boxed{r_E = 3963 \text{ miles}}$$

How close is your answer?

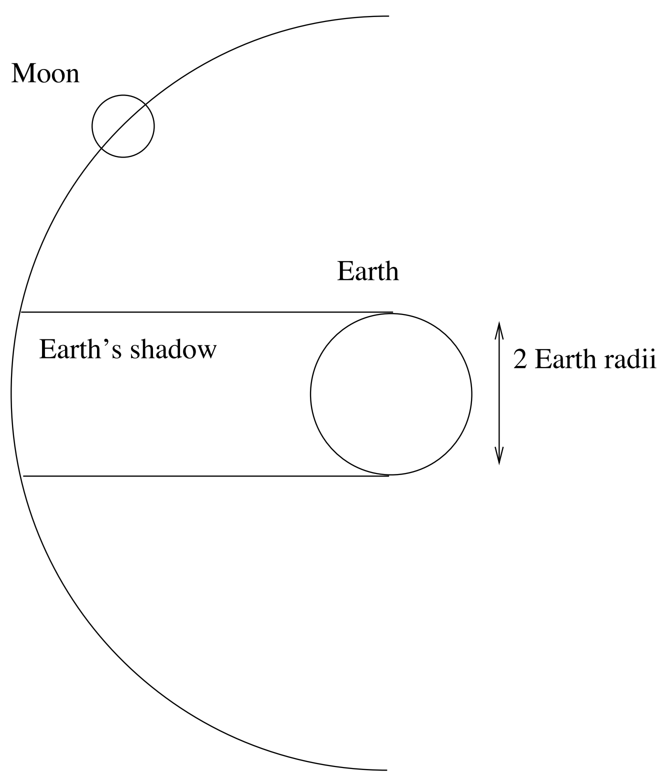
Second Step: the Moon

Knowing the radius of the Earth, Aristarchus (310-230 BCE) calculated the distance to the moon. How did he do it?

- Aristarchus knew that lunar eclipses were caused by the shadow of the Earth, which would be roughly two Earth radii in diameter. (This assumes the sun is very far away).

- He observed that a typical lunar eclipse lasted about 3 and 1/2 hours.

- It was also known that the moon takes 28 days to make a full rotation about the Earth.



Distance to the moon

- How many hours does it take for the moon to rotate about the Earth?

$$T = \underline{\qquad\qquad \text{hours} \quad}$$

- The moon travels through the earths shadow at in

$$t = \underline{\qquad 3.5 \text{ hours} \qquad}$$

- Assume the moon orbits the Earth in a giant circle, let R be the radius of this circe, (the distance from the center of the Earth to the center of the moon). The circumference of this circle is

$$C_R = \underline{\quad 2\pi \times R \quad} \approx \underline{\quad 6.28 \times R \quad}$$

- Let r_E be the radius of the Earth. If the moon moves around the Earth at a constant speed then the ratios are equal

$$\frac{C_R}{2r_E} = \frac{T}{t},$$

- Solving for R in terms of r

$$R \approx \underline{\qquad 61 \times r_E \qquad}$$

- Use the calculated radius of the Earth to find the distance to the moon.

$$R = \underline{\qquad\qquad\qquad}.$$

- The precise measured distance from the Earth to the moon is

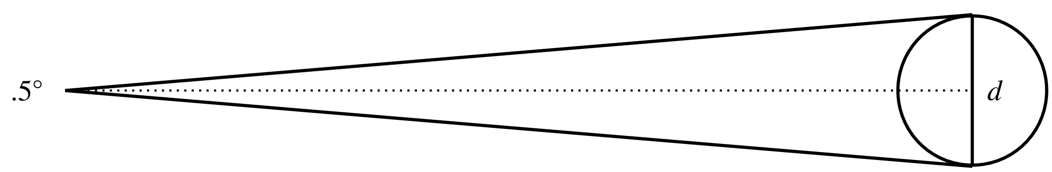
$$\boxed{R = 238,857 \text{ miles}}$$

How close is your answer?

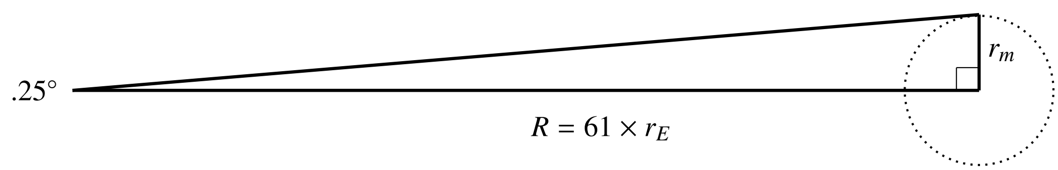
Radius of the moon

Knowing the distance to the moon, Aristarchus calculated the moon's raduis.

- The moon takes about 2 minutes (1/720 of a day) to set. The angular width of the moon is is about .5°.



- Divide the triangle in half:



- What is the ratio of the moon's radius r_m, to the Earth's radius, r_E.

$$\frac{r_m}{r_E} = \underline{\hspace{4cm}}.$$

- Using the Earth's radius calculate the radius of the moon,

$$r_m = \underline{\hspace{4cm}}.$$

- The calculated radius of the moon is about

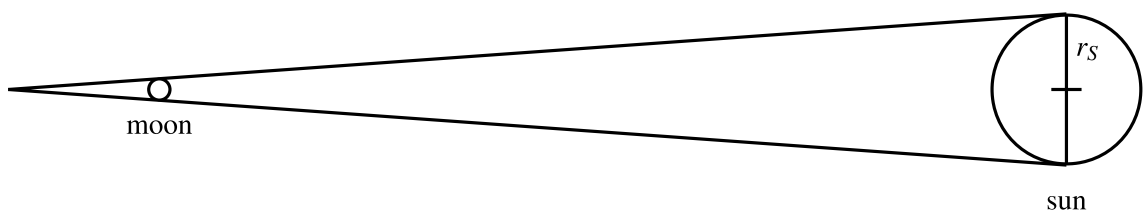
$$\boxed{r_m = 1080 \text{ miles}}$$

How close is your answer?

Third step: the Sun

- During a solar eclipse the moon completely covers the sun, so they have the same angular width.

- Aristarchus knew that the radius of the moon was about 1/180 the distance to the moon. Since the Sun and the moon have the same angular with he concluded the radius of the Sun is about 1/180 the distance to the Sun. (The correct answer is more like 1/215).



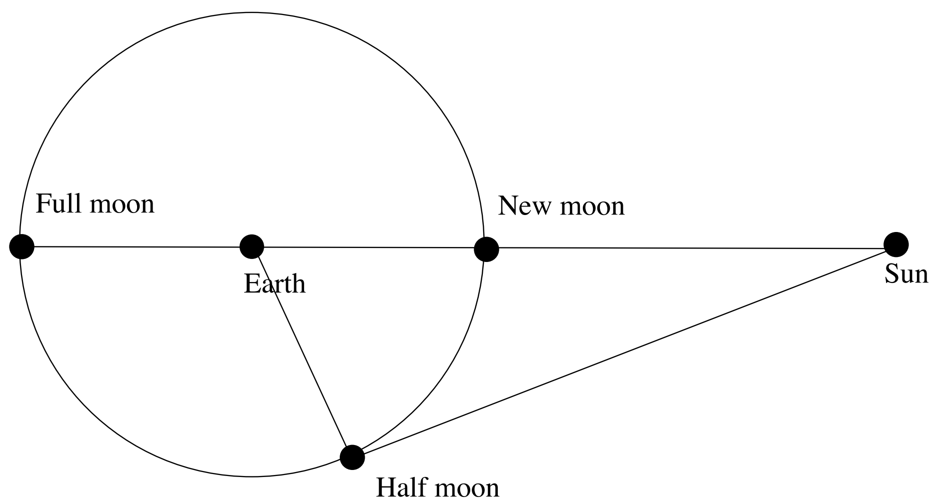
- So one needs just one of these measurements.

- Once again Aristarchus calculated the distance to the Sun.

Distance to the Sun

- Aristarchus knew that each new moon was one lunar month after the previous one.

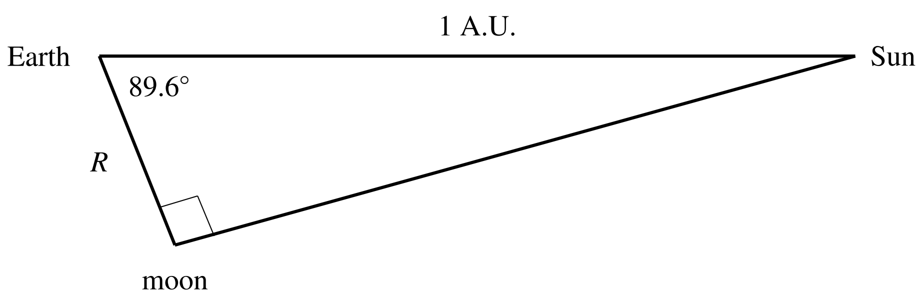
- By carful observation, he also knew that a half-moon occured slightly earlier than the midpoint between a new moon and a full moon; he measured the this discrepancy as 12 hours. (This is an unstable method as the true discrepancy is 1/2 an hour)



- By observing the shadow on the moon, he knew that a half moon occurred when the moon makes a right angle between the Earth and Sun.

Distance to the Sun

- We call the distance from the Earth to the Sun an **Astronomical Unit** (A.U.).
- Using the fact that a half moon occurs 12 hours, we calculate the angle between the moon and the Sun. This triangle is not drawn to Scale!



- Using the distance from the Earth to the moon, calculate the distance from the Earth to the Sun. This is known as an astronomical unit, A.U. How many of Earth's radii make an A.U.

$$\begin{aligned} 1\text{A.U.} &= \underline{\hspace{2cm}} \times R = \underline{\hspace{2cm}} \times r_E \\ &= \underline{\hspace{4cm}\text{miles}} \end{aligned}$$

- The actual value is

$$\boxed{1\text{A.U.} = 92,955,807 \text{ miles}}$$

- How close is your answer?
- What happens if you use 89° instead of 89.6° in your calculations.

Fourth step: the Speed of Light

- Galileo Galilei (1564-1642) attempted to find the speed of light. His experiment was the following. Have two poeple stand far away from him with a covered lanterns. One uncovers his lantern, then the other uncovers his lantern once the light is seen. Galileo said this experiment was inconclusive. But it did show that the speed of light was faster than the speed of sound.

- Ole Romer (1644-1710) and Chrstiaan Huygens (1629-1695) obtained a more accurate value for the speed of light, using a moon of Jupiter, Io.

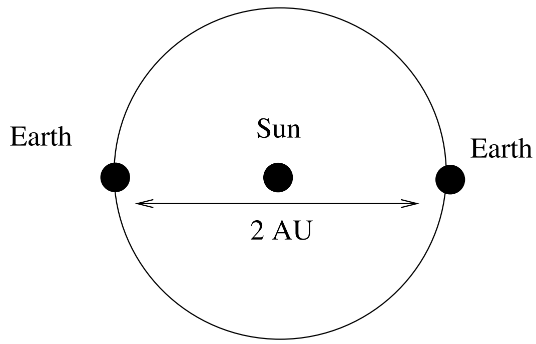
- Romer observed that Io rotated around Jupiter every 42.5 hours, by timing when Io entered and exited Jupiter's shadow.

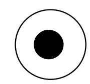
- He observed that this period was not uniform, in fact it lagged by about 20 minutes, depending on if Earth was aligned with Jupiter or opposed to Jupiter.

- Huygens concluded that it took light 20 minutes to travel 2 A.U.

The Speed of Light

- It takes light 20 minutes to travel 2 A.U.





- The speed of light, usually denoted c,

$$c = \frac{2\ \text{A.U.}}{20\ \text{min}} = \frac{1}{10}\frac{\text{A.U.}}{\text{min}} = \underline{\hspace{4cm}}\ \text{A.U.} / \text{sec.}$$

- What is the speed of light in mile per second

$$c = \underline{\hspace{4cm}}\ \text{mile/sec}$$

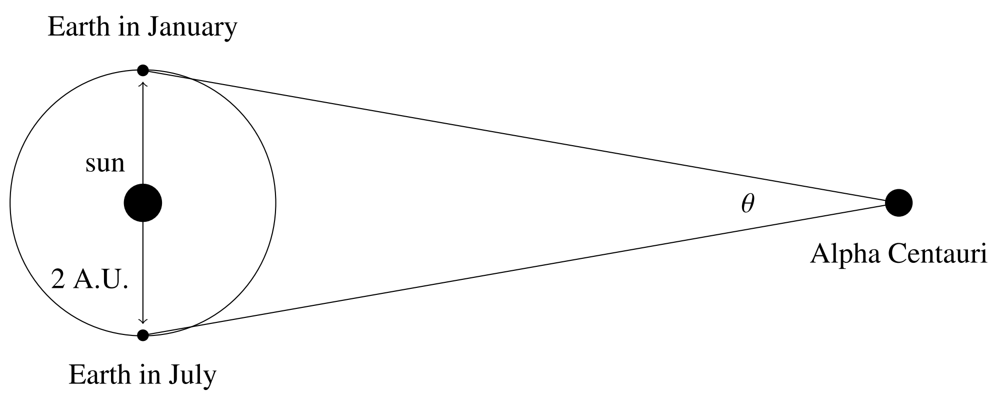
- The actual speed of light is

$$\boxed{c = 186,282\ \text{mile/sec}}$$

- How close is your answer?

Fifth step: the distance to nearby stars

- Knowing how large an A.U., one can obtain the distance to near by stars. This was first carried out successfully by Friedrich Bessel (1784–1846).
- The nearest visible star is Alpha Centauri.
- This is done by measuring the angle to the star on Earth at times six months apart. This process is known as triangulation or parallax.



- Because these distances are so vast, the angle θ is very, very small. For instance, for Alpha Centauri one has

$$\boxed{\theta = \frac{1}{4800}^{\circ} \;!!!}$$

- Since the distances are so great, we need an new unit of measurement, the light year. Alpha Centauri is about

$$\boxed{\text{4.3 light years}}$$

- By the way, Aristarchus proposed that the Earth circled the sun well before we knew this fact. However, the ancient Greeks dismissed his idea, because this would imply the stars a really, really far away (well... they are).