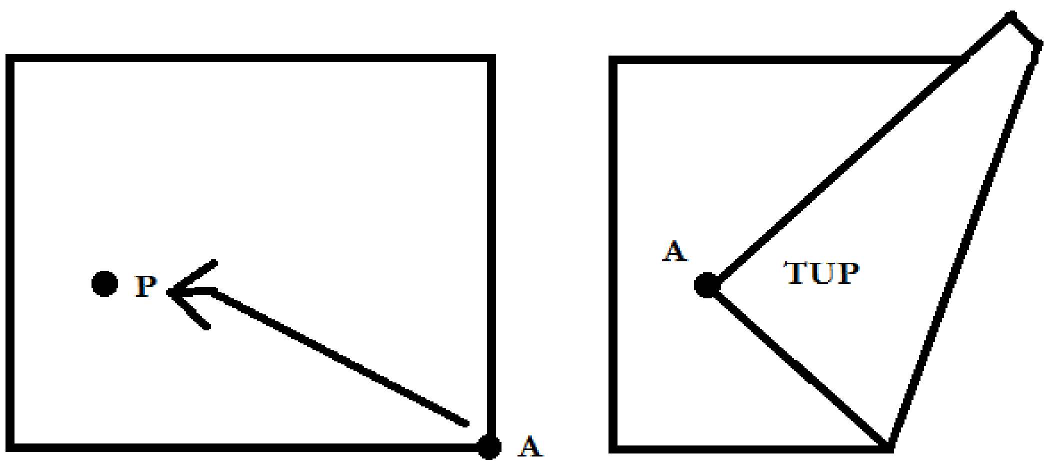
Haga's Origamics

Haga's “Origamics” are a series of activities designed to illustrate the science behind simple paper folding.

Activity I : TUPS (Turned-Up Parts)

Take a square piece of paper and label the lower right-hand corner A. Pick a random point on the paper and fold A to that point (see the picture). This creates a flap of paper, called a Turned-Up Part (TUP).

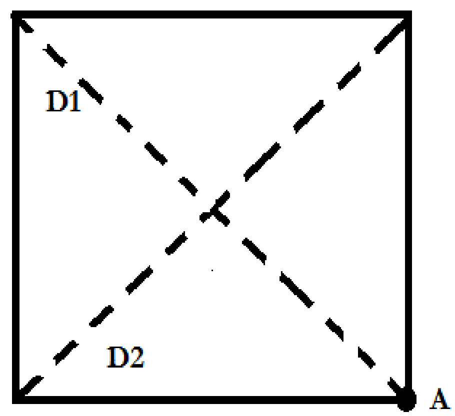


How many sides does your TUP have? Three? Four? Five? ______________________

Experiment with many TUP's to answer the following questions:

How many sides can a TUP have? (Provide an explanation as well)

Consider the following picture. Let D1 be the diagonal going through the point A and let D2 be the other diagonal.



If P lies on D1, what can you say about the TUP formed using P? Does it have 3 sides, 4 sides, etc?

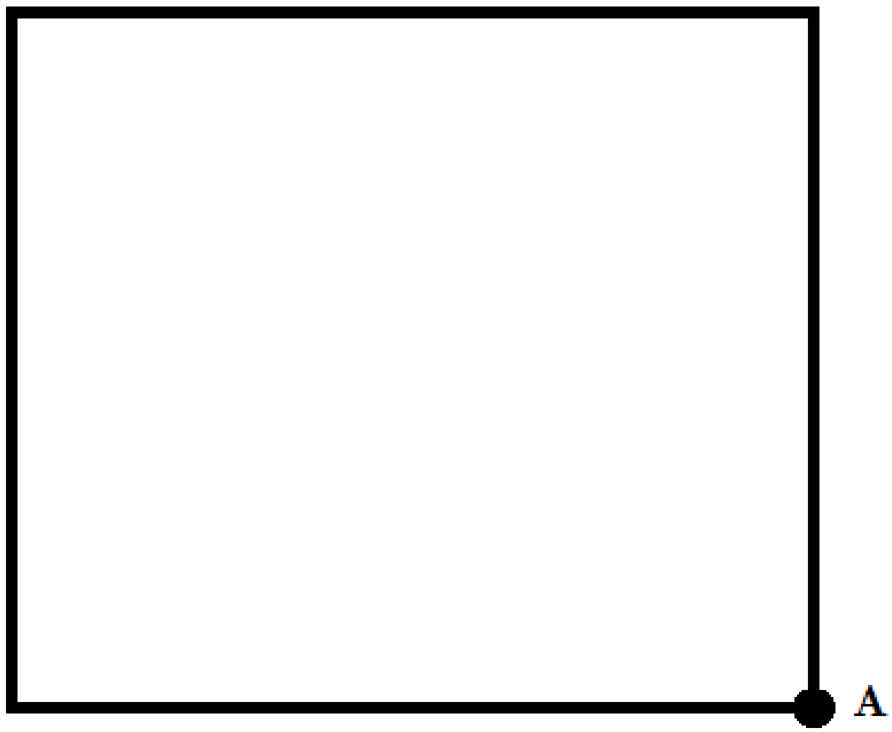
What about when P lies on D2?

When does P change from yielding 4 sides to yielding 3 sides?

Experiment with lots of points. Determine which of those points define TUPs with 3 sides and which define TUPs with 4 sides.

Make a conjecture about when a point defines a TUP with 3 sides and when a point defines a TUP with 4 sides.

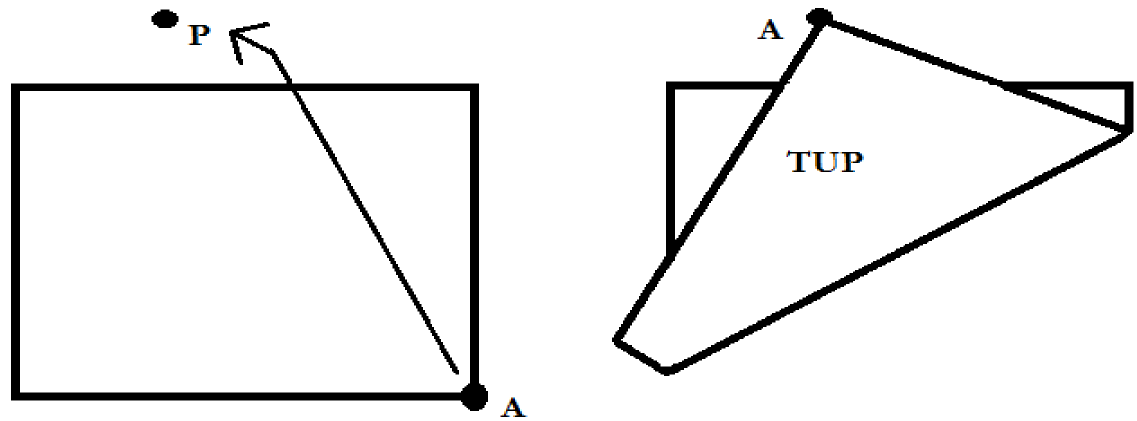
In the square below, draw and label the regions showing which points yield TUPs with 3 sides and which yield TUPs with 4 sides.



Explain how you found these regions:

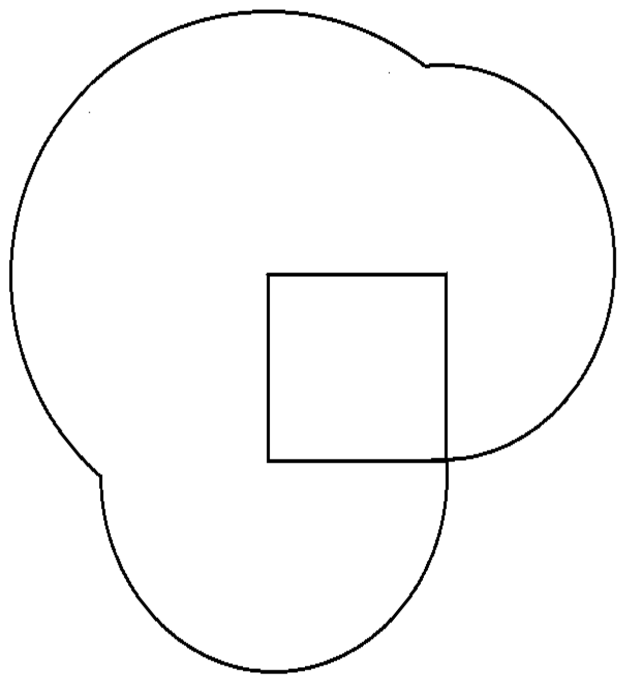
Consider the boundaries of your region. Do they yield TUP's with 3 sides or TUP's with 4 sides? Why?

So far, the point P had to lie within the square. What if P lies outside the square? For example we could fold a TUP as shown below:



What happens if P is really far away from the square?

Let's only look at P close enough to the square so that folding A to P does not just flip the square over. The region of points that we can fold to is drawn below. The original square is in the middle of the region.

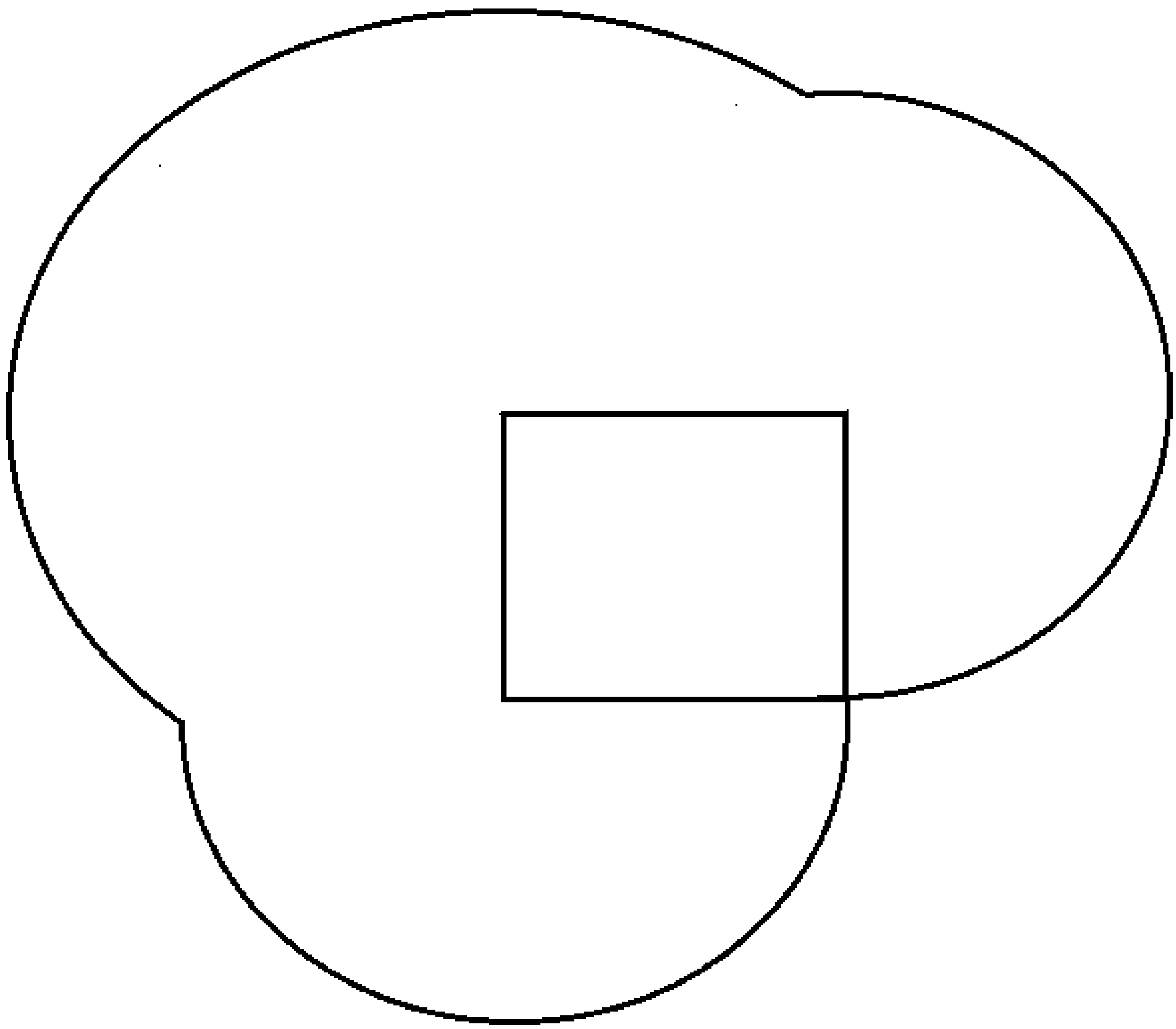


Experiment with different points. How many sides can TUP's obtained from points inside or outside the square have? Be sure to construct an example with each number of sides.

Experiment with lots of points. Determine which of those points define TUPs with 3 sides, 4 sides, and 5 sides.

Make a conjecture about when a point defines a TUP with 3 sides and when a point defines a TUP with 4 sides and when a point defines a TUP with 5 sides.

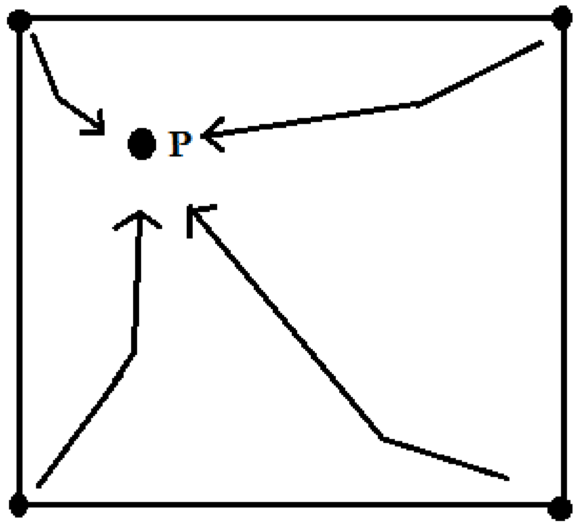
In the picture below, draw and label the regions showing which points yield TUPs with 3 sides, which yield TUPs with 4 sides, and which yield TUPs with 5 sides.

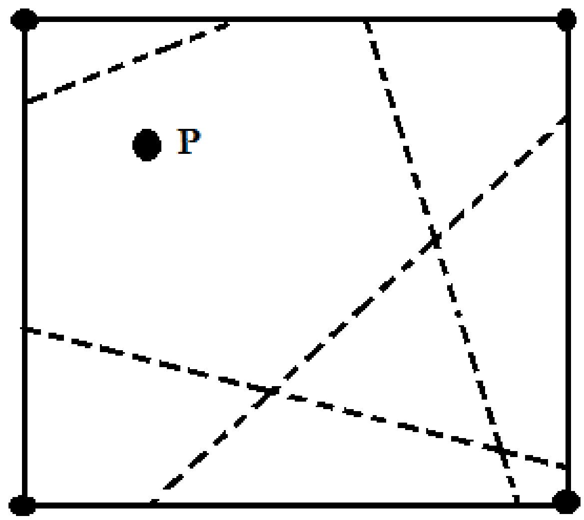


Explain how you found these regions. To which regions do the boundaries belong?

Activity 2 : All 4 Corners to a Point

Take a square sheet of paper and pick point P on it at random. Fold and unfold each corner to this point. The crease lines will make a polygon around P on the square (some sides of the square may be sides of a this polygon).





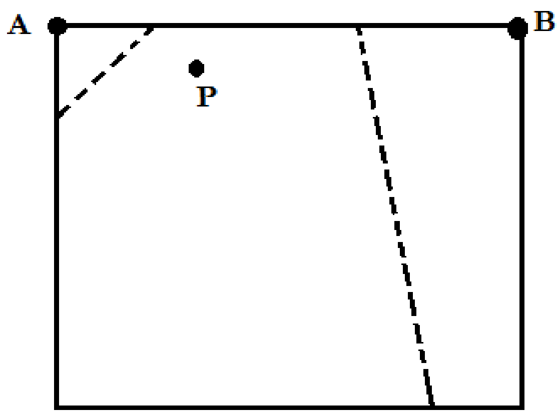
Experiment with many points to answer the following question:

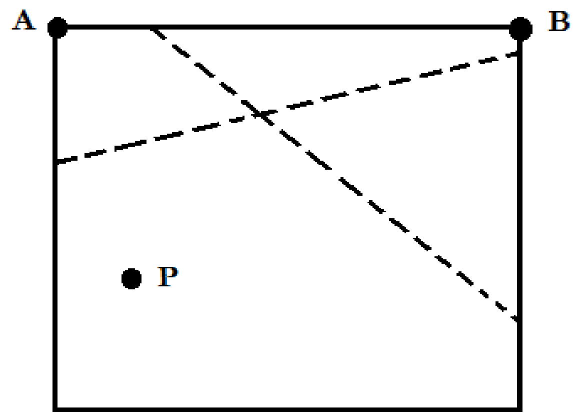
How many types of polygons are possible? Can you construct 3-sided, 4-sided, 5-sided, 6-sided, or 7-sided polygons?

Construct examples of each possible one. How do you know that these are the only types of polygons that can be constructed?

Let's first examine what happens on a single side.

Choose a point P and look at the crease lines gotten from folding points A and B to the point P. How many sides can these two creases contribute to the crease-polygon?



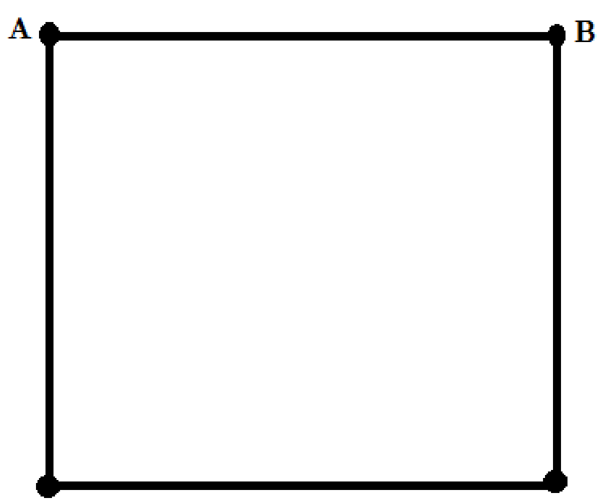


What happens if P is close to the top side?

What happens if P is close to the bottom side?

Experiment with lots of points to determine when the creases made by folding A and B to P yield two lines and when do they yield three lines.

In the square below, sketch the region(s) of points P where the creases made by folding A and B to P yield 2 lines and the region(s) of points P where the creases made by folding A and B to P yield 3 lines.



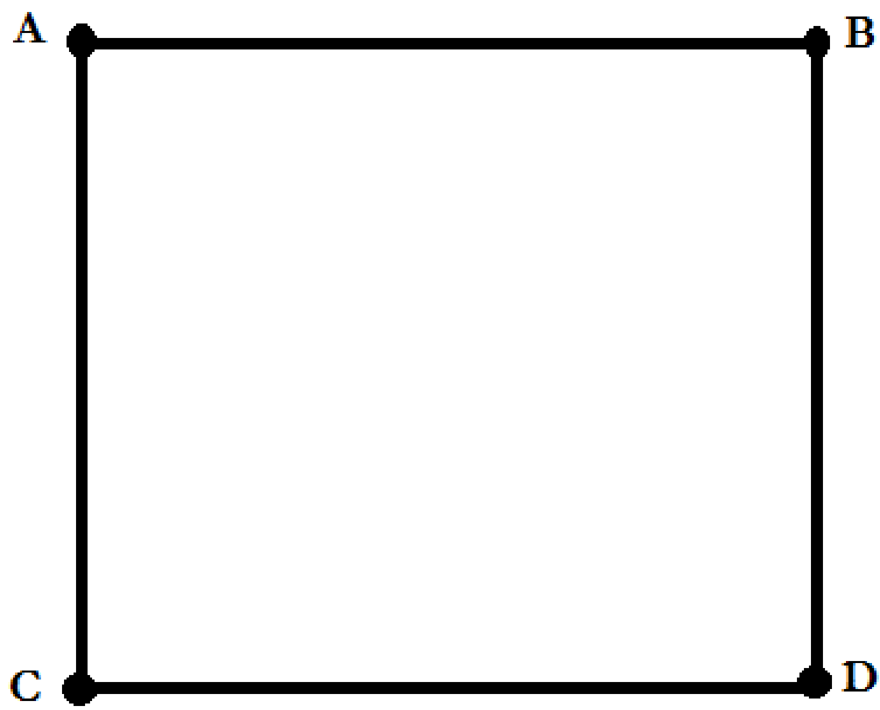
Can you form a similar picture for each side of the square?

In the square below, sketch the region(s) of points P where the creases made by folding A and C to P yield 2 lines and the region(s) of points P where the creases made by folding A and C to P yield 3 lines.

Do the same for the side with corners B,D.

And for the side with corners C,D.

Draw those regions all on the same square below.



Use that picture to answer the following questions:

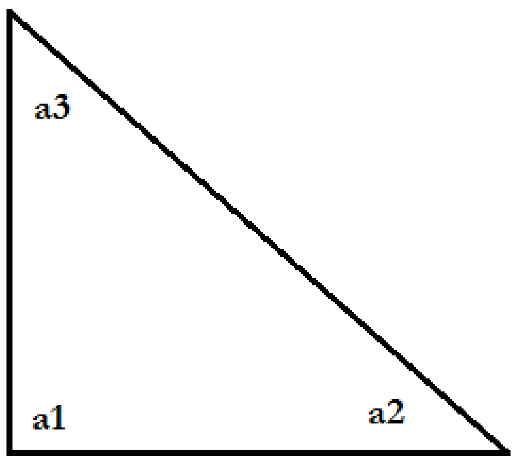
When does the crease-polygon associated to a point P have four sides?

When does the crease-polygon associated to a point P have five sides?

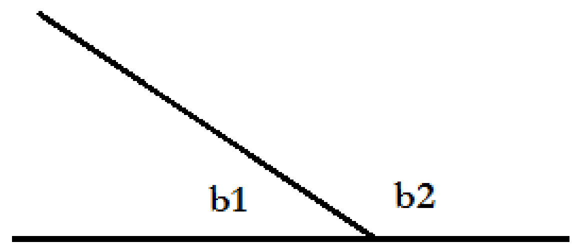
When does the crease-polygon associated to a point P have six sides?

Activity 3: Haga's Theorem

Recall the following facts about angles of a triangle and angles that form a straight line.



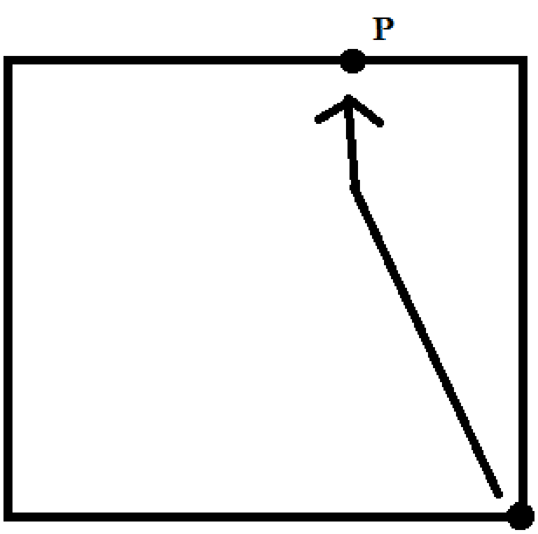
a1 + a2 + a3 = 180

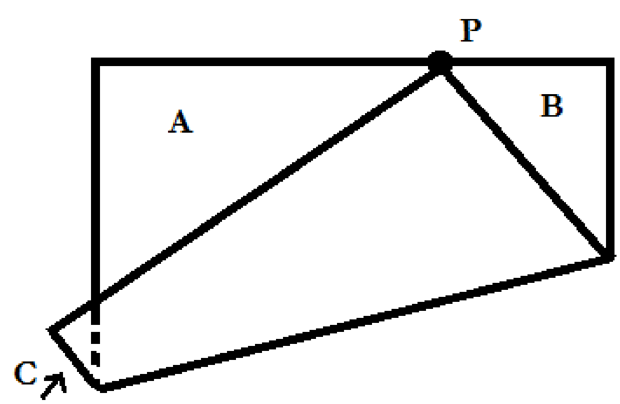


b1 + b2 =180

And, a1= 90° because it is a right angle.

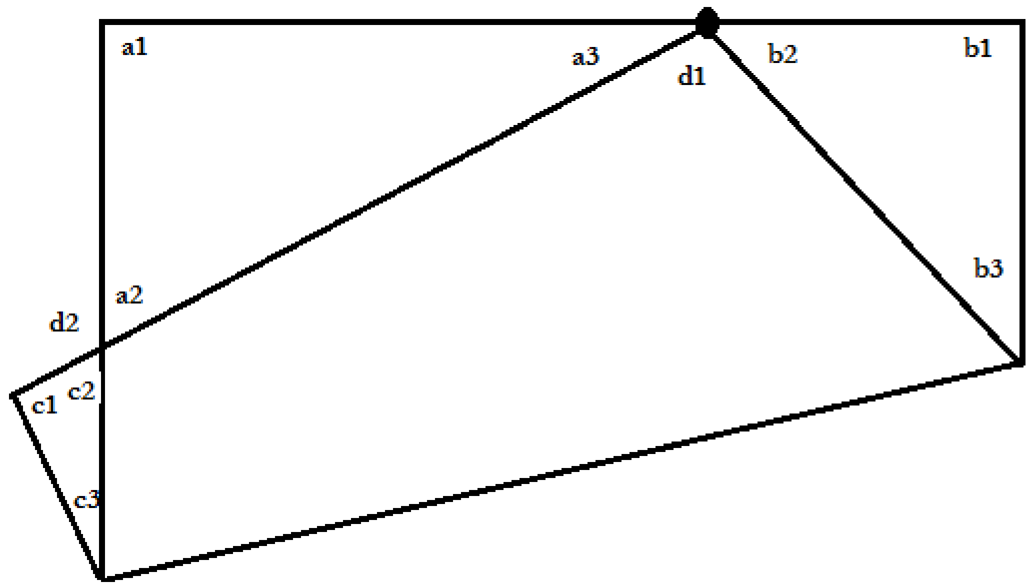
Take a square piece of paper and pick a random point P along the top edge of the paper.
Then fold the lower right corner to this point to make 3 triangles, A, B, and C.





Goal: Prove a nice relationship about the triangles A, B, and C.

Label the angles of A, B, C, and nearby angles as in the picture:



Using the facts about angles given in the beginning of the activity, write down everything we know about the angles in the figure.

For example, we know:

b1 = 90° (because it is one of the corners of the square, and so is a right angle)

a3 + d1 + b2 = 180° (because together they form a straight line)

Write all other possible equations about the angles in the figure: (use the next page too, if needed)

(extra room for angle equations)

These equations can tell us something about the relationship between the angles of triangles A, B, and C.

What is that relationship?

Use the equations from the previous page to show it.