3 Homework

Consider the recursion

A1. $f(1) = 2$

A2. $f(2) = 2$

B. $f(n+2) = f(n+1)f(n)$

X1. Compute the values $f(1), \ldots, f(6)$ for this recursion

X2. Prove that for every n in $\mathbf{N}$, the value $f(n)$ is a power of 2, namely a number of the form 2^p for some counting number p.

X3. Accepting that X2 is true allows us to write $f(n) = 2^{p(n)}$, where $p(n)$ is the counting number power of 2 giving $f(n)$. Write a table of values for $p(1), \ldots, p(6)$.

X4. Write the recursion for $p(n)$ from X3. Recognize it?

X5. Solve the recursion for $p(n)$ in X4, and thus solve the recursion for $f(n)$.

Fix numbers a, b, c and define $f(n) = an^2 + bn + c$ for all n in $\mathbf{N}$.

Y1. Put $a = 2$, $b = 0$, and $c = 1$. Compute the values $f(1), f(2), f(3), f(4)$.

Y2. Compute the successive differences $f(2) - f(1)$, $f(3) - f(2)$, and $f(4) - f(3)$ from the function values in Y1.

Y3. Find a formula for $f(n+1) - f(n)$ using the general a, b, c formula for $f(n)$. Simplify it as much as possible.

Y4. Use the formula from Y3 to write a recursion for $f(n)$.

Y5. Find values for a, b, c such that the recursion for $f(n)$ has the form

A. $f(1) = 1$.

B. $f(n+1) = f(n) + 2n + 1$

Hint: some of the numbers a, b, c could be zero.