

HEXAFLEXAGONS!

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(Places where templates came from are listed near the end.

You should go visit these sites to learn more about flexagons and get more templates to make these at home!)

1 Flexagons

Today, we are going to explore flexagons, and several of its variations. We'll start with something called the "TriHexaflexagon". Once you have made a working model and learn how to "flex" the model, you can start to think about the following questions.

Problem 1. *How many "sides" does this hexaflexagon have? (It may help to color the sides as you flex the hexaflexagon).*

Problem 2. *Can you make a diagram, or a map, of all the faces? For instance, if you are on face A, can you reach face B or C? A good starting point would be to put some labels on your page and then draw arrows to the sides you can reach from flexing.*

Now we're going to look at another hexaflexagon. This one is called a "hexahexaflexagon." (Why is it called this do you think? Think about it as you work on the following.)

Problem 3. *How many sides does the hexahexaflexagon have? (Again, color the sides).*

Problem 4. *For any side you start on, which faces you can reach by folding? Are there any that you can't reach? Make a list to help you keep track for each face.*

Problem 5. *Can you use the above information to make a diagram similar to the one you did above?*

Problem 6. *Using your diagram, can you determine the fastest way to get from one side to another?*

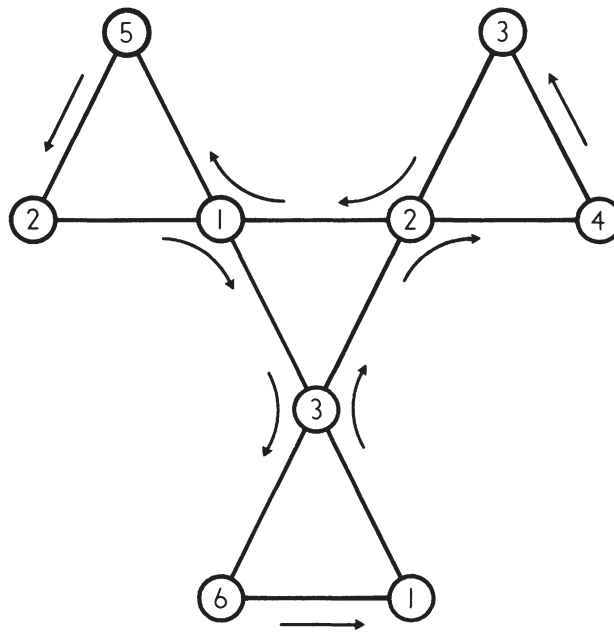


Figure 1: Diagram for the hexahexaflexagon, from Gardner's paper.

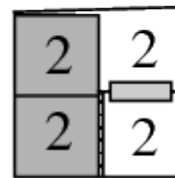
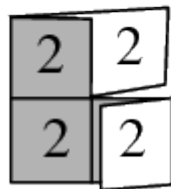
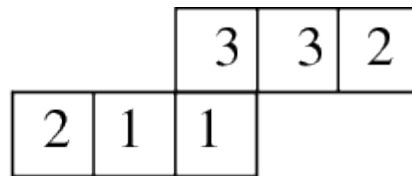
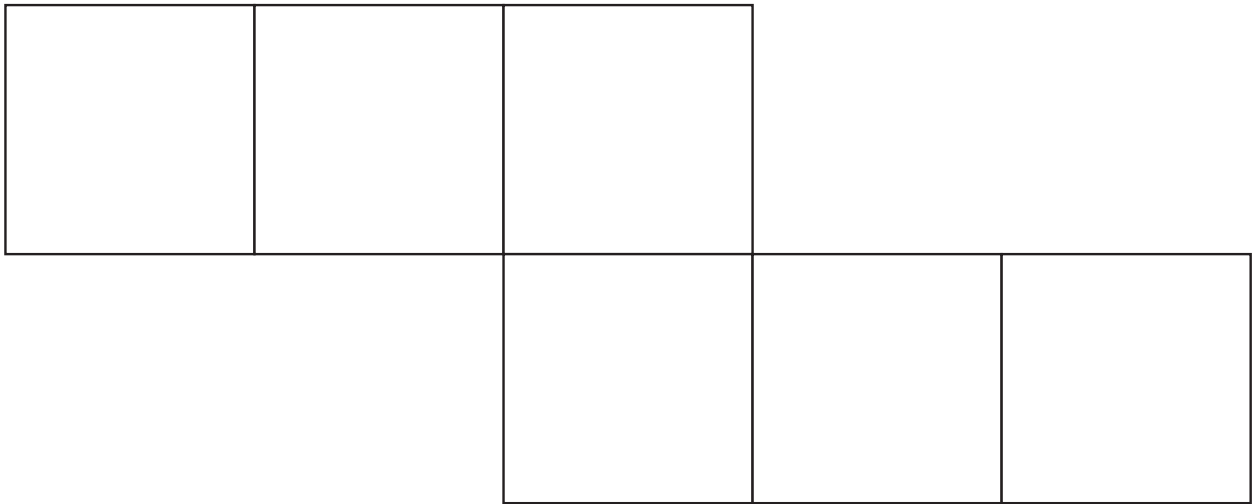
Problem 7. *It seems at times that you may encounter a side, say side 1 in the above picture, and you can normally reach side 5 by flexing, but after flexing a bit more you actually find that sometimes you can't reach side 5 by flexing from 1, and that flexing actually takes you instead to side 3! What is going on? Isn't this still the same side 1? (Maybe you can color a design around the center of such a side, and flex it around and see what happens?)*

There are many other types of flexagons (including more hexaflexagons!), with many different side lengths. On the last page are some links to websites with templates allowing you to make these other types of flexagons. We will now begin exploring some Tetraflexagons.

2 Tetraflexagons

There are many other types of flexagons out there. The hexaflexagon is the most popular, but you may have seen toys based off the following which are also easy to make from paper. They are called tetraflexagons (What does this name tell you about the flexagon? What does "tetra" mean?)

Below are templates to make several different types of tetraflexagons. Build them and try to answer some of the questions we've been asking about hexaflexagons (how many sides, trying to diagram the different sides and how they fold, etc.). Pay particular attention to what happens based on if you flex along different creases either horizontally or vertically. How should we name these tetraflexagons judging by how we named the hexaflexagons?

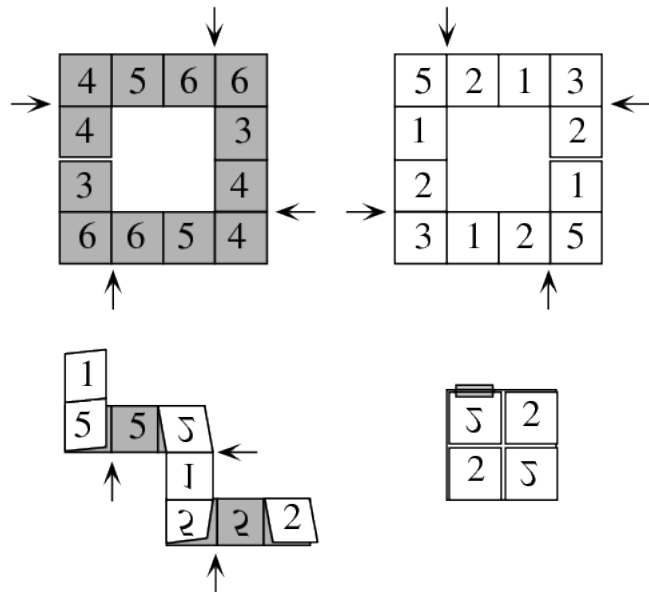
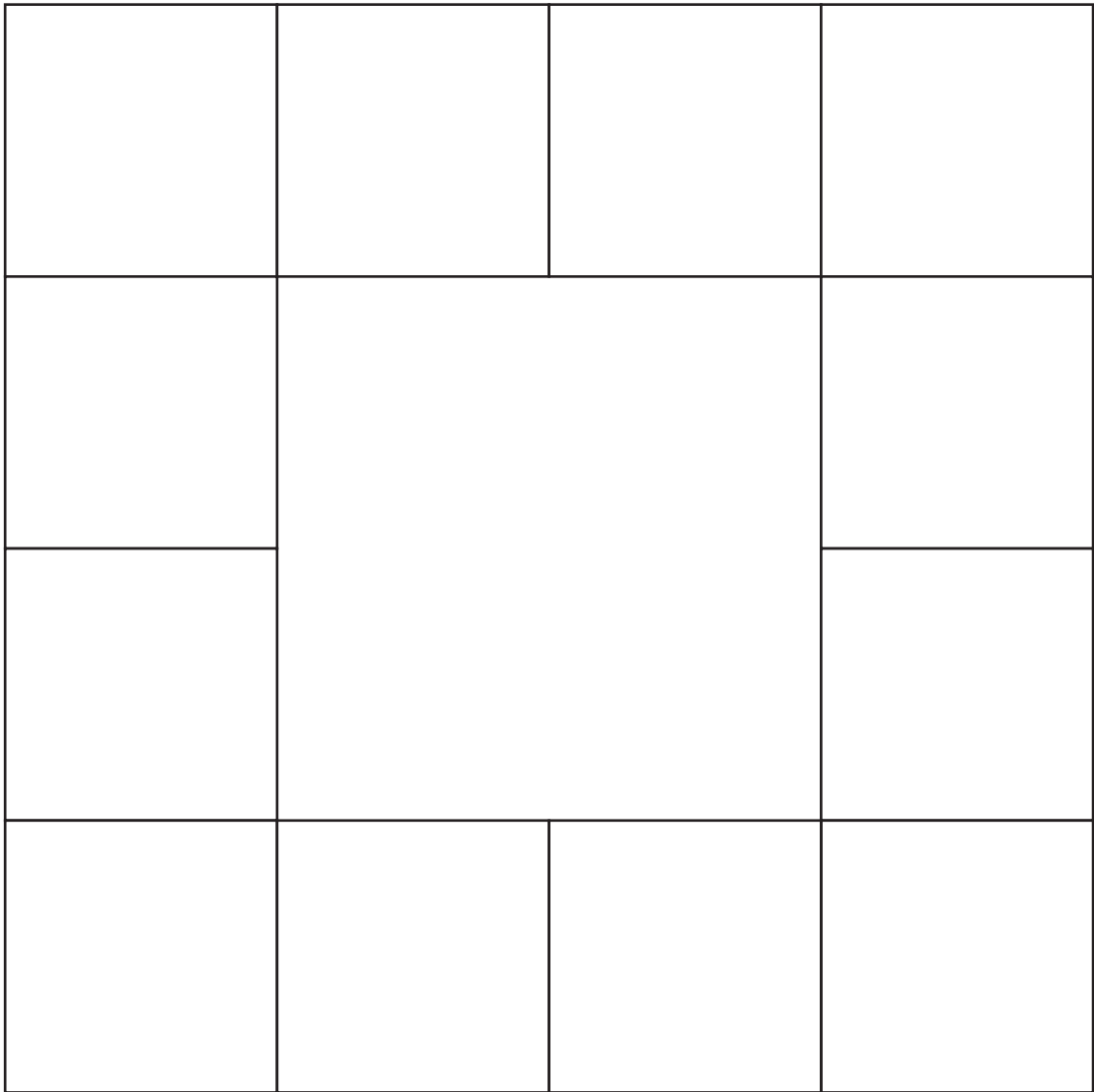


1	1	2	3
3	2	1	1
1	1	2	3

4	4	3	2
2	3	4	4
4	4	3	2

	1	1	2
3	3		2
	1	1	2

1	1
1	1
1	1



3 Webpages for more info and templates

Paper by Martin Gardner explaining Hexaflexagons:

http://maa.org/pubs/focus/Gardner_Hexaflexagons12_1956.pdf

*Website where the templates we used came from **as well as some awesome videos:***

<http://www.puzzles.com/hexaflexagon/>

Awesome website with printable templates and instructions for other hexaflexagons with 3, 4, 5, 6, or 7 sides: <http://www.flexagon.net/>

Here where I got the instructions for the tetraflexagons:

<http://mathworld.wolfram.com/Tetraflexagon.html>

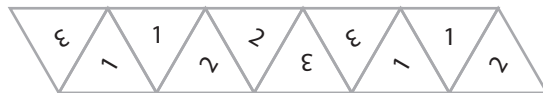
SIMPLE INSTRUCTIONS FOR FOLDING A TRIHEXAFLEXAGON

1. CUT OUT THE BLANK TEMPLATE AT THE LEFT, AND CREASE EACH OF THE LINES.

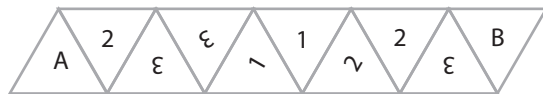
2. ORIENT THE TEMPLATE AS SHOWN:



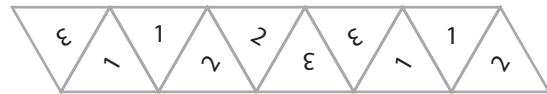
3. WRITE NUMBERS ON IT, EXACTLY AS SHOWN, WITH THE NUMBERS TILTED WHERE SHOWN:



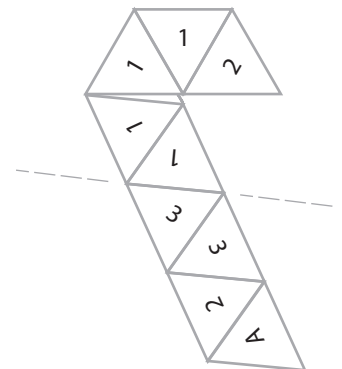
4. FLIP THE TEMPLATE OVER FROM TOP TO BOTTOM, SO THE TRIANGLE AT THE LEFT (WITH THE 3 ON IT) STAYS TO THE LEFT. LABEL THE SECOND SIDE AS SHOWN:



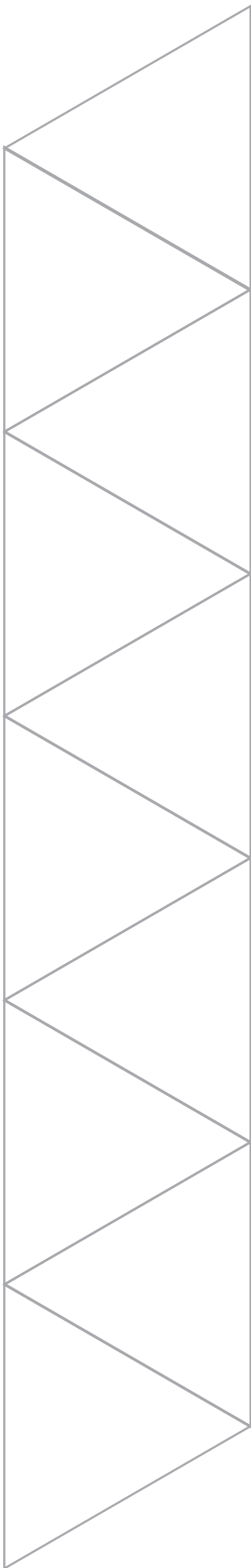
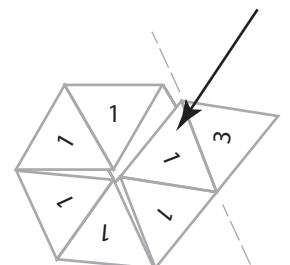
5. TURN THE TEMPLATE BACK TO THE FRONT AND FOLD IT ONCE AS SHOWN. THE LEFT SIDE SHOULD GO **BEHIND** THE RIGHT SIDE, AS SHOWN IN THE ILLUSTRATION IN STEP 6:



6. FOLD THE BOTTOM PART OF THE STRIP UP AND **AWAY** FROM YOU, AS SHOWN. BE SURE THE NUMBERS LOOK LIKE THE ILLUSTRATION AT THE RIGHT. NOTE THAT THE "1" FACE GOES **IN FRONT OF** THE "2" FACE. (SEE ARROW BELOW). THE FLEXAGON SHOULD NOW LOOK LIKE THE LAST ILLUSTRATION BELOW. **IT SHOULD NOW HAVE ALL "1" S SHOWING, AND ONE TAB WITH A "3"!!**



7. **LAST STEP:** FOLD TAB 3 AWAY FROM YOU. IT HAS A LETTER "A" ON BACK. GLUE OR TAPE FACE "A" TO FACE "B" IN BACK, AND YOU HAVE A TRIHEXAFLEXAGON!

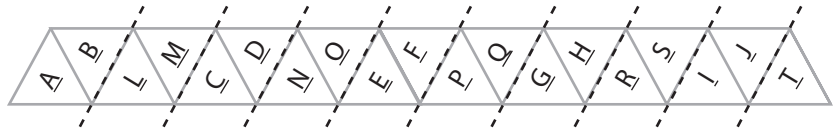


HEXAHEXAFLEXAGON BLANK TEMPLATE

INSTRUCTIONS

1. CUT OUT THE TEMPLATE & LIGHTLY LABEL IT WITH PENCIL AS SHOWN BELOW. LATER, YOU CAN ERASE THE PENCIL AND DECORATE YOUR FLEXAGON.

FRONT

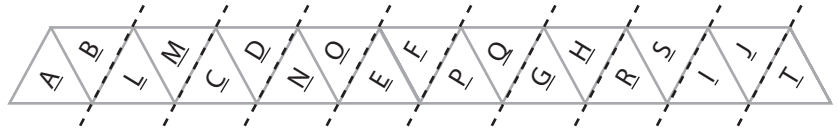


BACK



2. MAKE MOUNTAIN FOLDS ALONG THE LINES SHOWN BELOW:

FOLDS



3. NOW FOLD THE STRIP SO THAT IT LOOKS LIKE THE DIAGRAM BELOW:

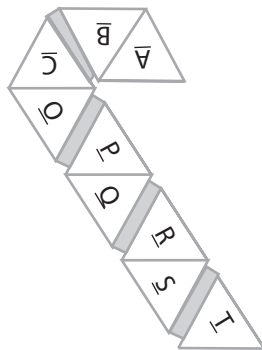
FRONT



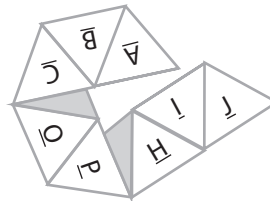
BACK



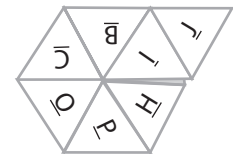
4. NOW FOLD YOUR STRIP IN TO THE SHAPE SHOWN HERE:



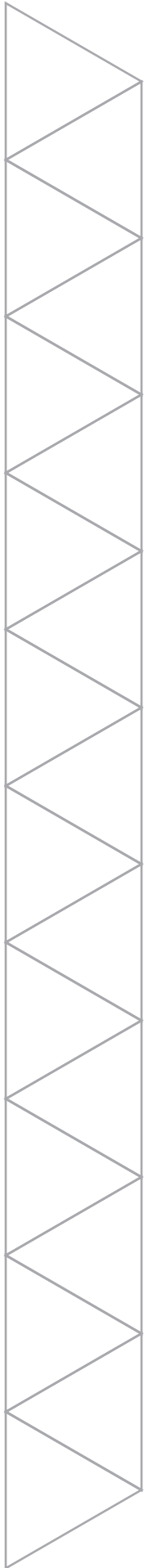
5. ...AND FOLD AGAIN LIKE SO:



6. BEND THE J FLAP AWAY FROM YOU & GLUE FACE I TO FACE K.



...AND NOW YOU HAVE A WORKING HEXAHEXAFLEXAGON!



Gathering for Gardeners

TRIHEXAFLEXAGON
BLANK
TEMPLATE



TO ASSEMBLE
USE "SIMPLE INSTRUCTIONS FOR
FOLDING A TRIHEXAFLEXAGON" PDF
<http://www.puzzles.com/hexaflexagon>