Gaussian Numbers

Integers

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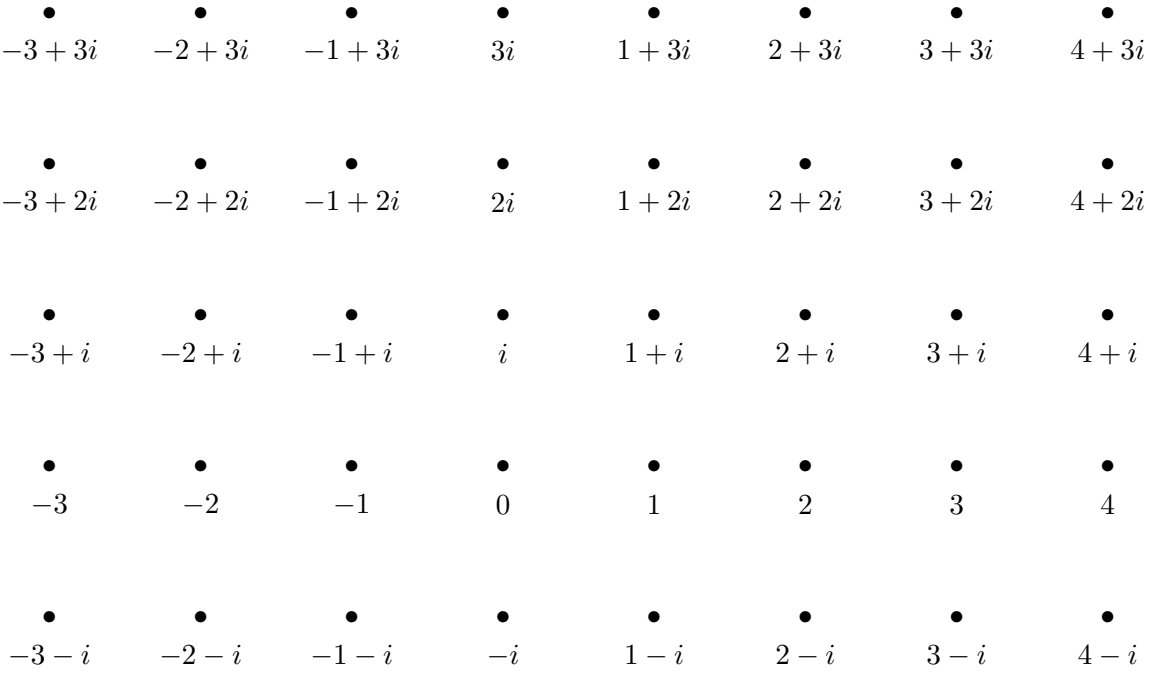

Addition of Gaussian integers

Examples:

$$(1+i)+1=2+i$$

$$(1+i)+i=1+2i$$

$$(1+i)+(1+i)=2+2i$$

1. Add $1+i$ and $1-i$.

2. Connect the points
(a) 0 and $1+i$;
(b) 0 and $1-i$;
(c) 0 and $(1+i)+(1-i)$;

3. Add $1 + 2i$ and 1.

4. Connect the points
(a) 0 and $1 + 2i$;
(b) 0 and 1;
(c) 0 and $(1 + 2i) + 1$;

Multiplication of Gaussian Integers

Examples:

$$1 \times 1 = 1$$

$$1 \times i = i$$

NOTE:

$$i \times i = -1$$

5. Multiply i and $(1 + i)$

6. Connect the points
(a) 0 and $1 + i$;
(b) 0 and $i \times (1 + i)$;

7. Multiply i and $(2 + i)$

8. Connect the points
(a) 0 and $2 + i$;
(b) 0 and $i \times (2 + i)$;

9. Multiply $1 + i$ and $(1 - i)$

10. Connect the points
(a) 0 and $1 + i$;
(b) 0 and $1 - i$;
(b) 0 and $(1 + i) \times (1 - i)$.

11. Multiply $2 + i$ and $(2 - i)$

12. Connect the points
(a) 0 and $2 + i$;
(b) 0 and $2 - i$;
(b) 0 and $(2 + i) \times (2 - i)$.

13 Multiply $3 + 2i$ and $(3 - 2i)$

14. Connect the points
(a) 0 and $3 + 2i$;
(b) 0 and $3 - 2i$;
(b) 0 and $(3 + 2i) \times (3 - 2i)$.

Prime Numbers

A positive integer p is a prime number if it is divisible only by 1 and by itself.

15. List the prime numbers less than 30.

Square Numbers are

$$1 \times 1 = 1$$
$$2 \times 2 = 4$$
$$3 \times 3 = 9$$
$$\ldots$$

16. List the square numbers up to 200.

17. Which prime numbers can be written as a sum of two square numbers? (Hint: Try the prime numbers up to 30)

18. Compute the absolute value of $2 + i$ squared

$$|2 + i|^2 = (2 + i)(2 - i).$$

How is this absolute value related to 2^2 and 1^2?

(b)Compute the absolute value of $3 + 2i$ squared

$$|3 + 2i|^2 = (3 + 2i)(3 - 2i).$$

How is this absolute value related to 2^2 and 3^2?

(c) Compute the absolute value of $4 + i$ squared

$$|4 + i|^2 = (4 + i)(4 - i).$$

How is this absolute value related to 4^2 and 1^2?)

19. Compute the products and their absolute values squared:
Multiply $1 + i$ and $2 + i$
Multiply $1 - i$ and $2 + i$
Multiply $2 + i$ and $2 + 3i$.
Multiply $2 - i$ and $2 + 3i$.
Multiply $2 + i$ and $4 + i$.
Multiply $2 - i$ and $4 + i$.
Multiply $3 + 2i$ and $4 + i$.
Multiply $3 - 2i$ and $4 + i$.

20. For which two prime numbers can we express their product as a sum of two square numbers?

(Hint: Use questions 18 and 19. Try to express a product of two prime numbers less than 20 as a sum of two square numbers.)

21. In how many ways can we express the product of two prime numbers as a sum of two square numbers?

22. (Difficult problems) In how many ways can you express $5 \times 13 \times 17$ as a sum of two squares? How about $3 \times 5 \times 13$?