Rules of Chomp
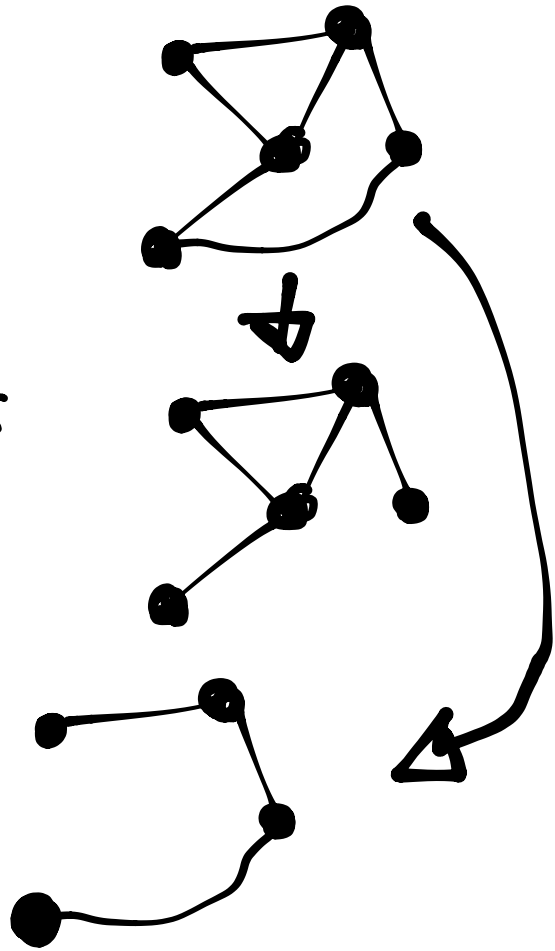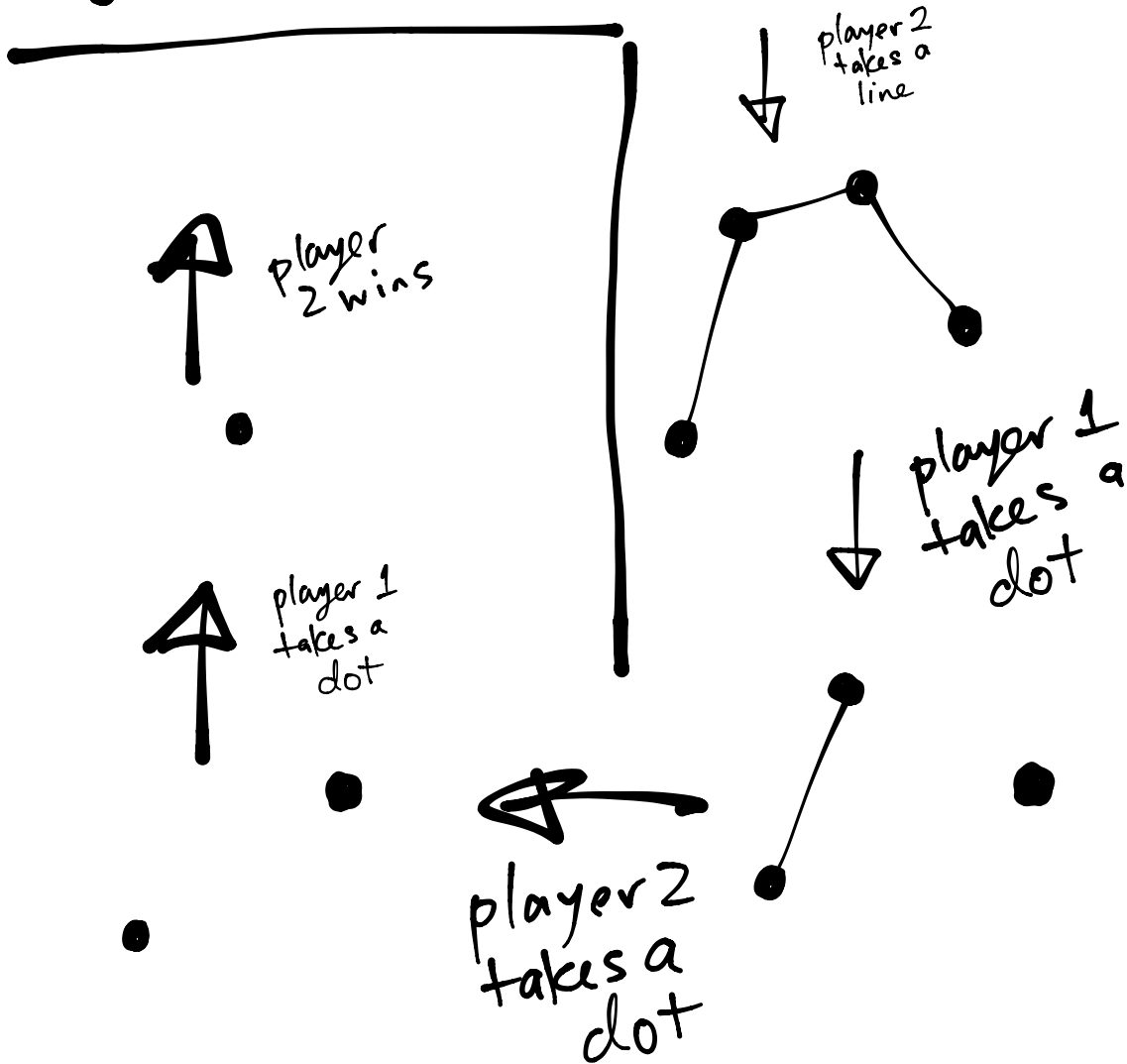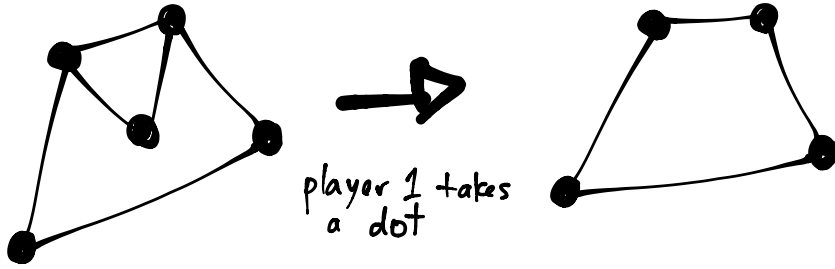
- You are given a graph which consists of dots and lines

- Players take turns either removing a single line

  or removing a dot and all connected lines

- The winner is the player to remove the last dot

EXAMPLE GAME $^{(2)}$

player 1 takes
a dot

player 2
takes a
line

player 1
takes a
dot

player 2
takes a
dot

player 1
takes a
dot

player
2 wins

GAME 1
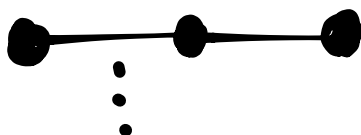
●————●————●————●

GAME 2

●————●————●————●————●

QUESTIONS

- IS THERE A STRATEGY FOR PLAYER 1 TO ALWAYS WIN GAME 1?

  WHAT ABOUT PLAYER 2?

- WHAT ABOUT GAME 2?

GAMES 1 & 2
USE __LINEAR__
__GRAPHS:__

●———●

●———●———●
    ⋮

_____

__QUESTION__:

Is there a
strategy so
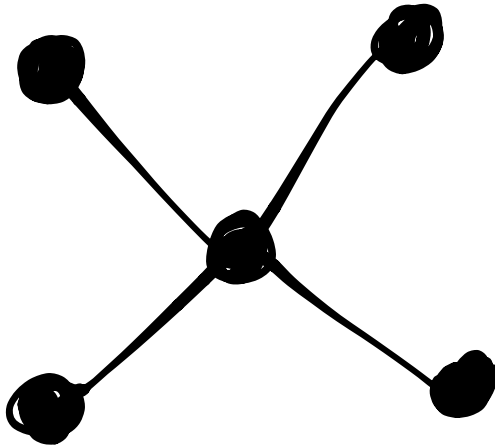that player 1
can win on any
linear graph?

ANSWER:

PLAYER 1
REMOVES THE
MIDDLE DOT
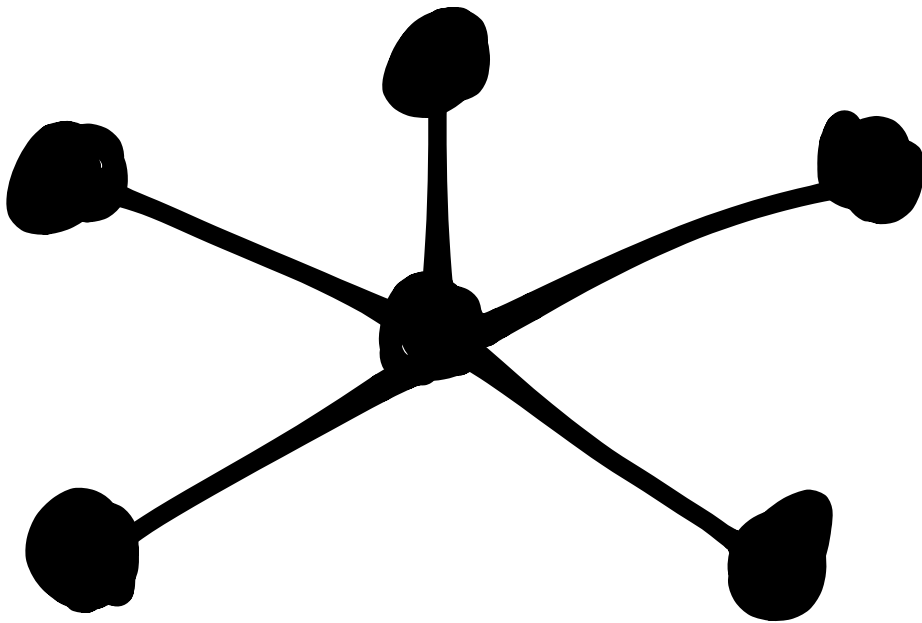 (IF ODD # OF DOTS)
OR MIDDLE LINE
(IF EVEN # OF DOTS)

THEN, PLAYER 1
MIMICS PLAYER 2
ON THE REMAINING
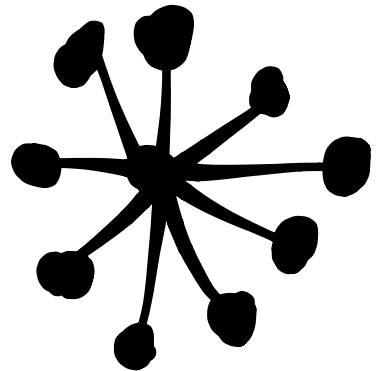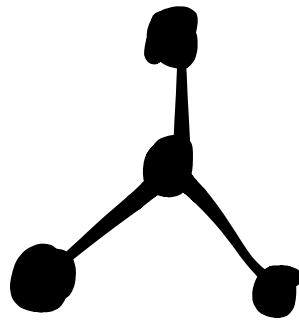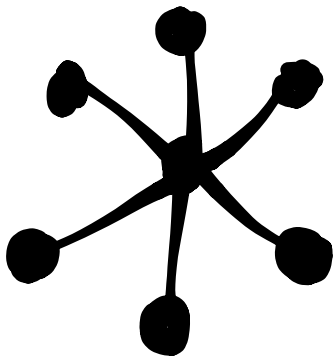2 IDENTICAL
 GRAPHS.

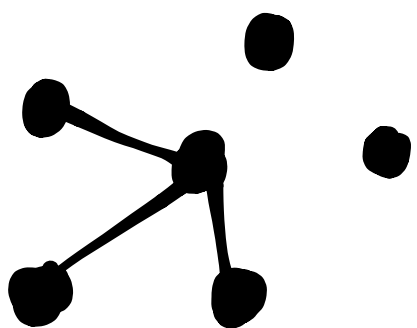# GAME 3

GAME 4

GAMES 3 & 4
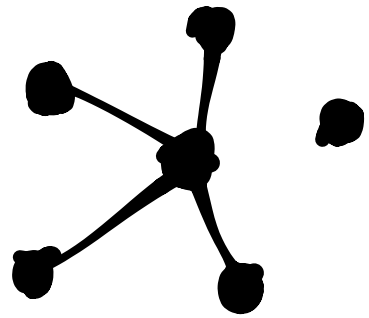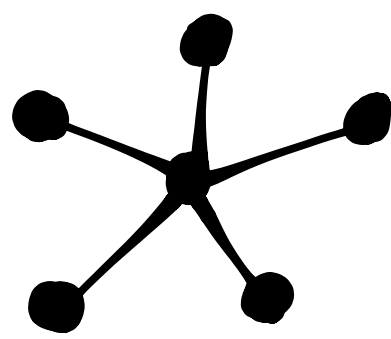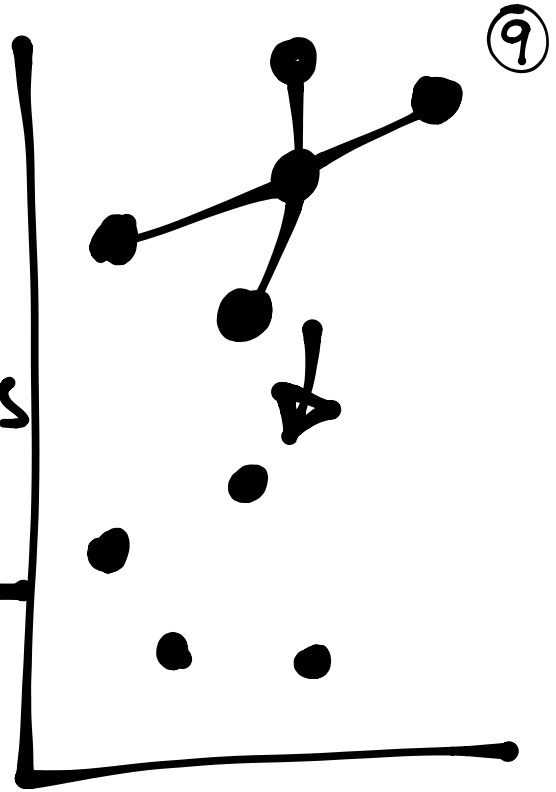# USE "STAR" GRAPHS

QUESTION: Is

THERE A STRATEGY
SO THAT PLAYER 1
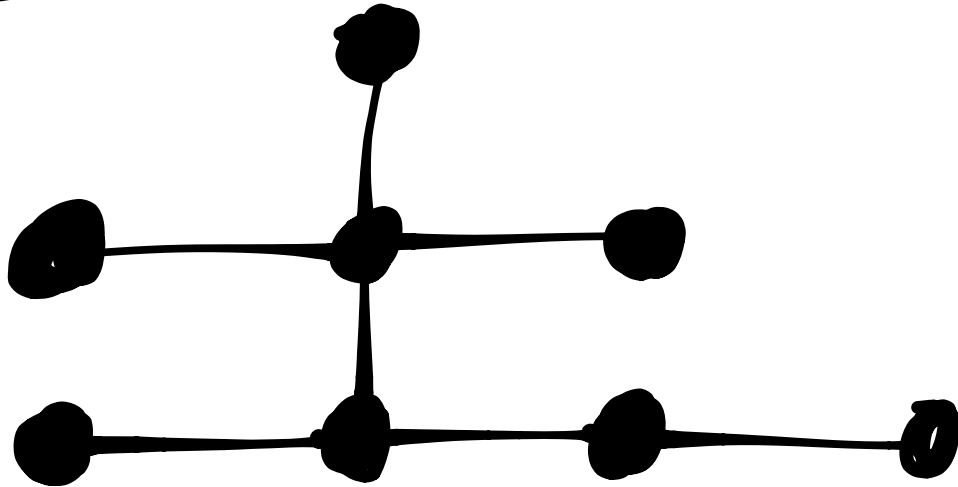ALWAYS WINS WITH
A STAR GRAPH?

ANSWER:

IF ODD # OF
DOTS,
PLAYER 1 REMOVES
CENTER DOT

IF EVEN # OF
DOTS,
YOU CAN ALWAYS
FORCE YOUR
OPPONENT TO
HAVE EVEN #
DOTS
& LINES

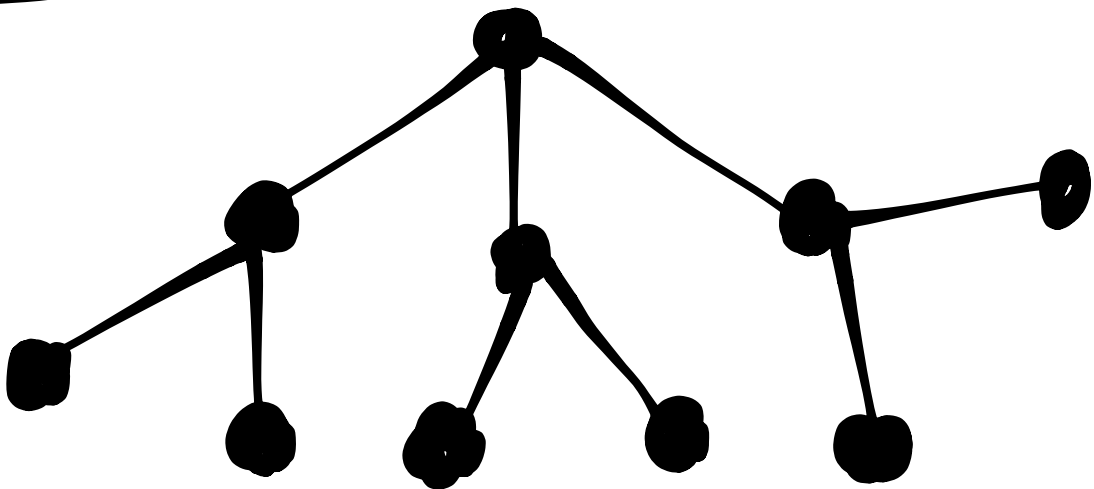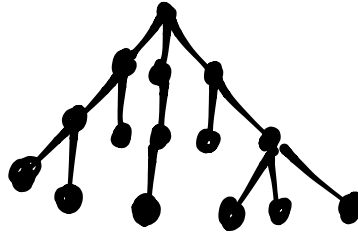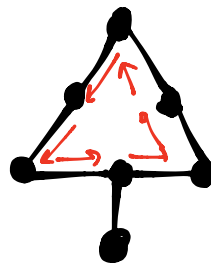# GAME 5

GAME 6

ALL GRAPHS SO
FAR HAVE BEEN
"TREES"
— MEANING
THE GRAPH IS
CONNECTED AND
CONTAINS NO
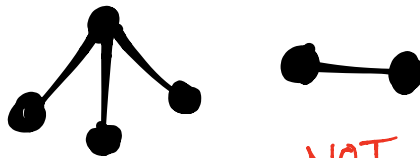LOOPS OR CYCLES

TREE:

NOT A TREE:

HAS A
LOOP!

NOT A TREE:

NOT
CONNECTED!

QUESTION:
CAN PLAYER 1
ALWAYS WIN
CHOMP ON A TREE?

STRATEGY FOR PLAYER $\underline{1}$ TO WIN ON A TREE:

ALWAYS FORCE OPPONENT TO HAVE EVEN # of DOTS AND LINES
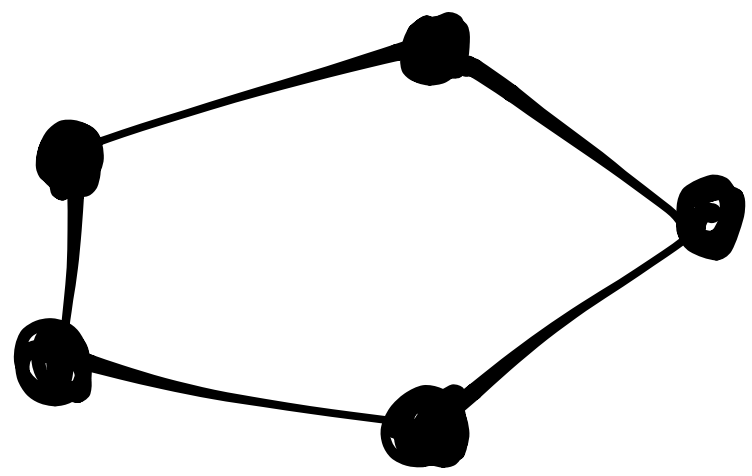
CAN THIS ALWAYS BE DONE?

IMPORTANT HINTS:

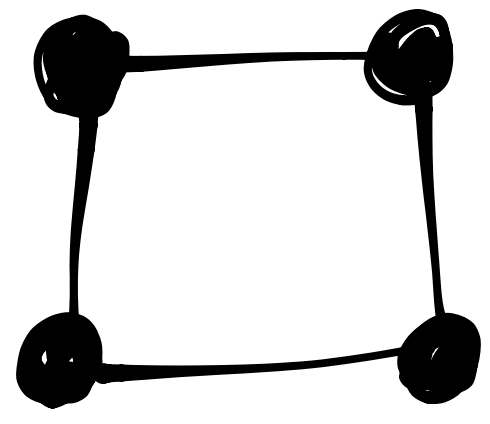● ON A TREE,
#DOTS = #LINES + 1   (WHY?)

● ON ANY GRAPH WITH ODD #DOTS, SOME DOT HAS EVEN # OF LINES ATTACHED   (WHY?)
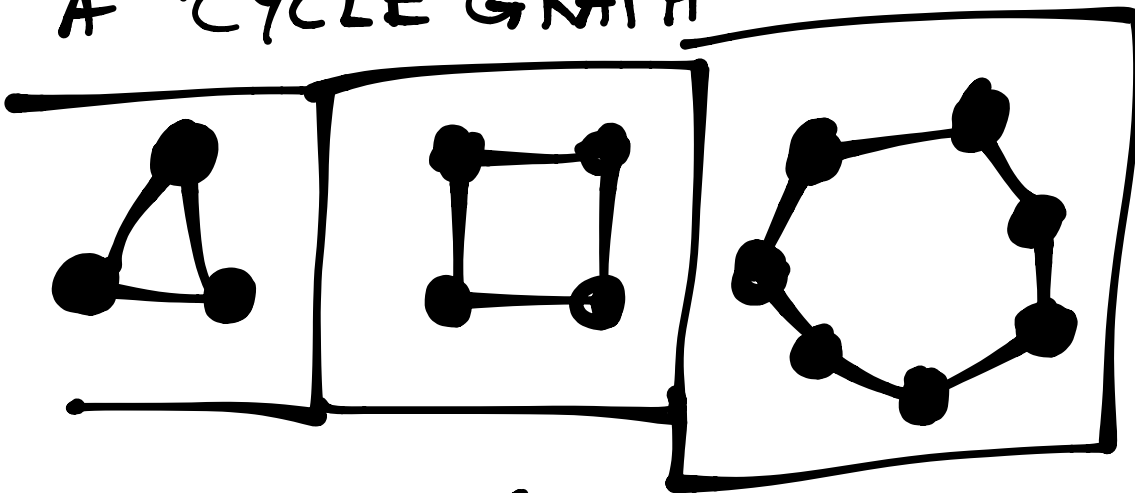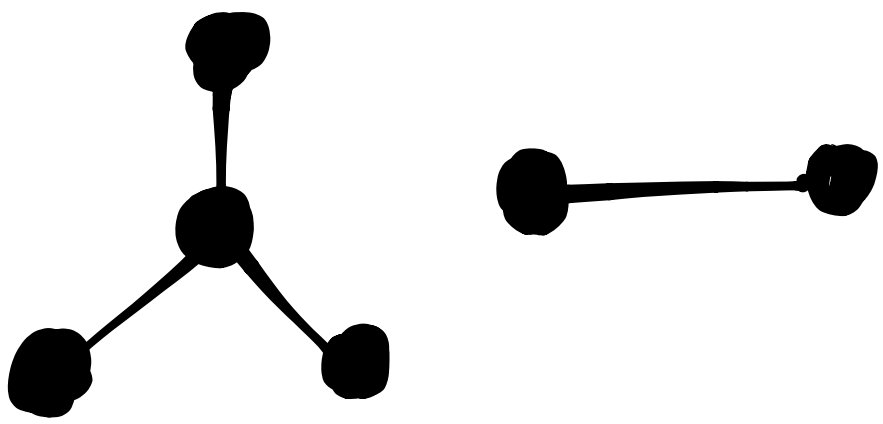
GAME 7

GAME 8

QUESTION:

DOES PLAYER 1 HAVE A STRATEGY TO ALWAYS WIN GAMES 7 & 8? WHAT ABOUT PLAYER 2?
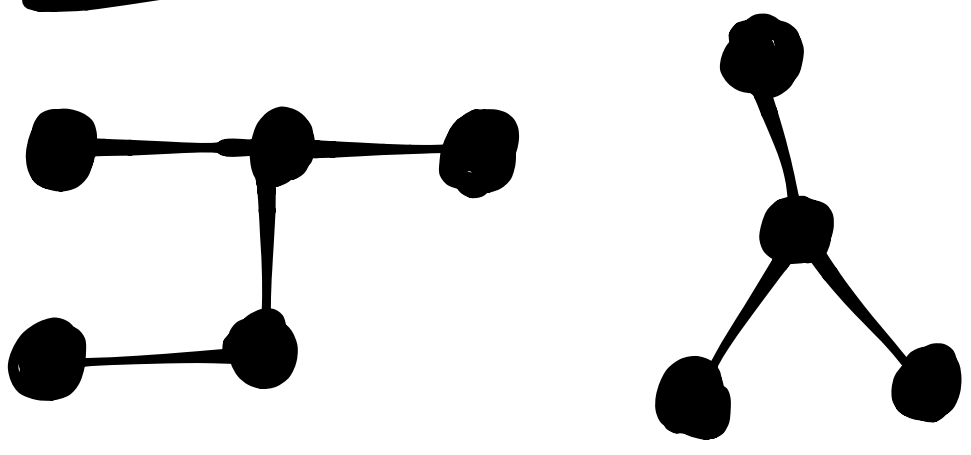
GAMES 7 & 8 USE A "CYCLE GRAPH"

WHICH PLAYER CAN ALWAYS WIN CHOMP ON A CYCLE?
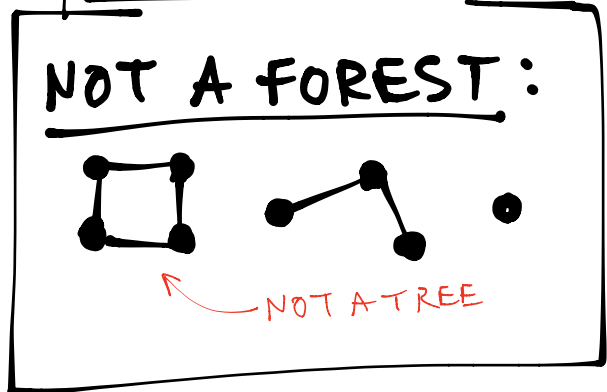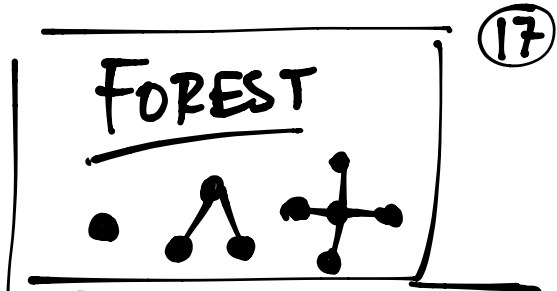
GAME 9

GAME 10

WHICH PLAYER
HAS A STRATEGY
TO ALWAYS
WIN GAME 9?
  GAME 10?

POTENTIAL STRATEGY:

ALWAYS FORCE
YOUR OPPONENT TO
HAVE EVEN # DOTS
AND EVEN # LINES

GAMES 9 & 10
USE GRAPHS
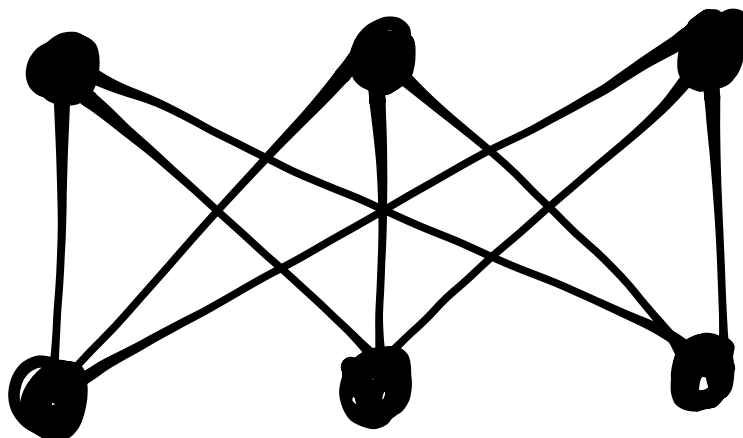CALLED "FORESTS"
— MEANING
SEVERAL SEPARATE
TREES

FOREST

NOT A FOREST:

← NOT A TREE

WHICH PLAYER ALWAYS HAS
A WINNING STRATEGY
PLAYING CHOMP ON A FOREST?

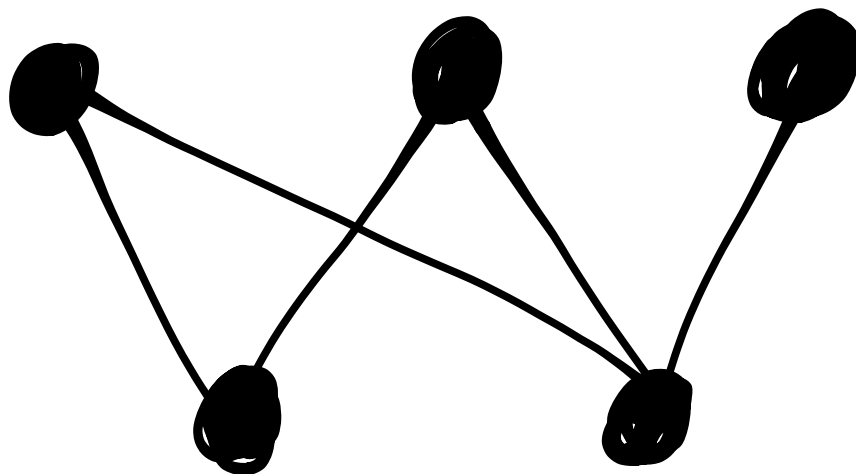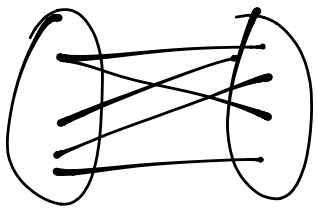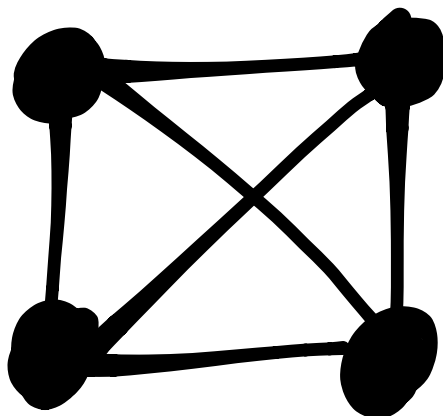|	EVEN # DOTS	ODD # DOTS
EVEN # LINES	?	?
ODD # LINES	?	?

GAME 11:

GAME 12:

GAMES 11 & 12
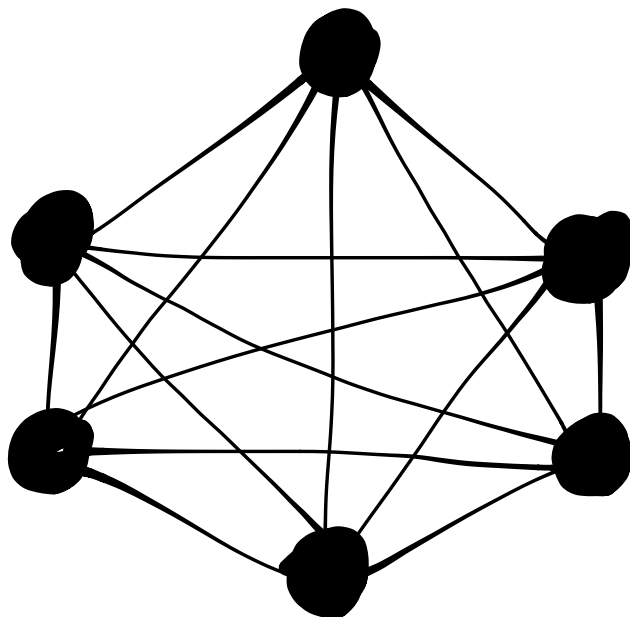USE "BIPARTITE"
GRAPHS:

DOES THE STRATEGY
FROM BEFORE WORK?

WHICH PLAYER HAS
A WINNING STRATEGY
WITH A BIPARTITE
GRAPH?

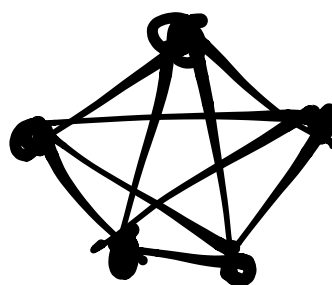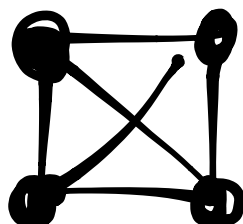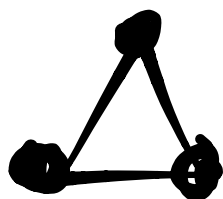|	EVEN # DOTS	ODD # DOTS
EVEN # LINES		
ODD # LINES		

GAME 13:

GAME 14:

GAMES 13 & 14 USE COMPLETE GRAPHS:

PLAYER 1 ALWAYS HAS A WINNING STRATEGY ...
    UNLESS
#DOTS = MULTIPLE OF 3