

SPROUTS!

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Sprouts is a very simple game to play with 2 people, but far more interesting to ask math questions about! We'll go over how to play Sprouts, then pose a few questions for you to think about and experiment with today.

How To Play Sprouts: Begin by drawing any number of dots on the page. Pick a starting player. The first player either draws a line connecting two dots on the page or connecting a single dot to itself. Then the player adds one dot along the line somewhere. However, you **must** follow the following rules:

- The line may be straight or curved, but must not touch or cross itself or any other line.
- The new spot cannot be placed on top of one of the endpoints of the new line. Thus the new spot splits the line into two shorter lines.
- No spot may have more than three lines attached to it. For the purposes of this rule, a line from the spot to itself counts as two attached lines and new spots are counted as having two lines already attached to them.

The last one is arguably the most important. If a spot has 3 lines coming from it, you cannot draw a line to it. Once a player draws a line and adds a dot, then the opposing player

The last player who is able to make a move wins the game!

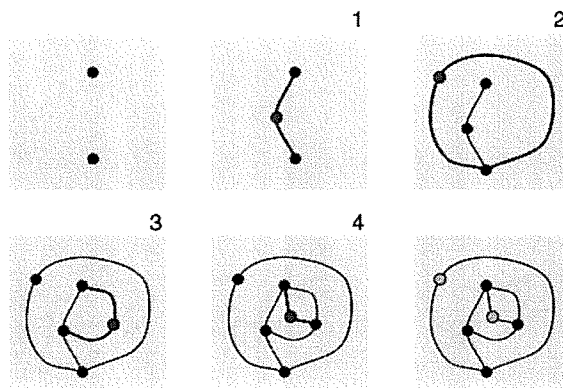


Figure 1: A game of Sprouts beginning with two spots. (image from Wikipedia)

Play a few games with people at your table, maybe varying the number of starting dots, and then think about how to answer the following questions.

Questions to Think About:

1. Does the game ever end? If so, how long does a game last? What is the longest number of turns a game can last? What is the shortest? (It might help to pick a certain number of starting dots, then play around with that starting number until you think you have an answer, then try it for another starting number of dots.)
2. Pick a number of starting dots. Can you come up with a winning strategy for the first player?
3. If you feel like you have, or are bored with Sprouts here are some variants. First, you can just change the rule of Sprouts so that the last player to make a move is the *loser* instead of the winner. Try answering the above questions with this variant.
4. Another variant is called "Brussels Sprouts". It is similar to Sprouts, and in most ways easier to find answers to.

You begin with a number of crosses. Then players take turns drawing a line connecting two edges of a cross and then adding a short mark along the edge they just drew. We can ask the same sorts of questions as we did for Sprouts. Pick a number of starting crosses. Does a game always end? What's the shortest number of moves needed to end a game? What's the longest? Who wins the game?

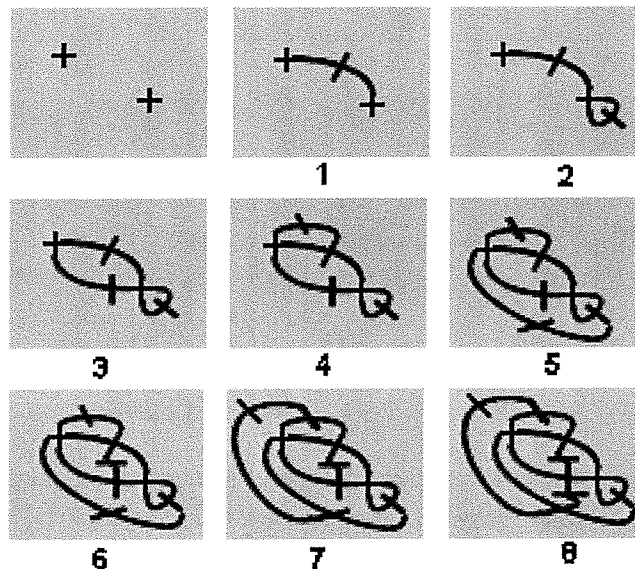


Figure 2: A game of Brussels Sprouts beginning with two crosses. (image from Wikipedia)