Dissections

1/24/2016

A **dissection** of a polygon is a decomposition of the polygon into finitely many polygons (called **pieces**). In the figure below, the triangle A and quadrilateral B are dissected into triangles. The pentagon and the hexagon are each dissected into four pieces.

Problem 1

Draw some quadrilaterals, pentagons, hexagons, heptagons, and octagons, and dissect them into triangles.

Problem 2

Can any polygon with N sides be dissected into triangles for all values of N?

Two polygons A and B are congruent by dissection if A can be dissected into pieces A_1, A_2, A_3, ..., A_n, and B can be dissected into pieces B_1, B_2, B_3, ..., B_n such that $A_1 \cong B_1$, $A_2 \cong B_2$, ..., $A_n \cong B_n$, (where $\cong$ means **congruent** to).

The square and the L-shaped hexagon in the above Figure are congruent by dissection.

Property: Two polygons that are congruent by dissection have the same area.

Problem 3

Suppose right triangle ABC ($\angle ABC = 90°$) and rectangle $DEFG$ have the same area and that $AB = DE$. Show that they are congruent by dissection.

Problem 4

Suppose obtuse triangle ABC ($\angle ABC > 90°$) and rectangle $DEFG$ have the same area and that $AB = DE$. Show that they are congruent by dissection.

Problem 5

Suppose acute triangle ABC and rectangle $DEFG$ have the same area and that $AB = DE$. Show that they are congruent by dissection.

Problem 6

Suppose rectangle $ABCD$ has side lengths $AB = CD = 12$ and $BC = AD = 3$. Show that $ABCD$ is congruent by dissection to a square whose side is 6.

Problem 7

Suppose rectangle $ABCD$ has side lengths $AB = CD = 9$ and $BC = AD = 4$. Show that $ABCD$ is congruent by dissection to a square whose side is 6.

Problem 8

Suppose rectangle $ABCD$ has side lengths $AB = CD = 25$ and $BC = AD = 4$. Show that $ABCD$ is congruent by dissection to a square whose side is 10.

Problem 9

Show that any rectangle is congruent by dissection to a square of the same area.

Problem 10

In the figure below, the hexagon $ABCDEF$ is comprised of two adjacent squares $ABGF$ and $CDEG$. Show that $ABCDEF$ is congruent by dissection to a square.

Problem 11

Three-dimensional dissection of a polyhedron is defined analogously to a polygon dissection (each piece of the dissection must be a polyhedron). Show that a $4 \times 5 \times 6$ rectangular prism is congruent by dissection to a $3 \times 5 \times 8$ rectangular prism.

Problem 12

Show that a $3 \times 25 \times 45$ rectangular prism is congruent by dissection to a $15 \times 15 \times 15$ cube.

Problem 13

Show that a $24 \times 25 \times 45$ rectangular prism is congruent by dissection to a $30 \times 30 \times 30$ cube.

Problem 14

Show that any rectangular prism is congruent by dissection to a cube of the same volume.

Problem 15

Show that any two polygons with the same area are congruent by dissection!