

The Mathematics of Billiards

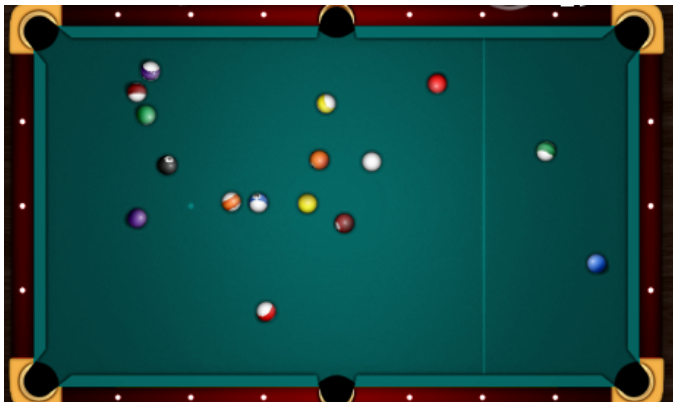
Washington University Math Circle

Chris Cox

March 6, 2016



One thing you could do but we
won't: play real billiards! ☹️



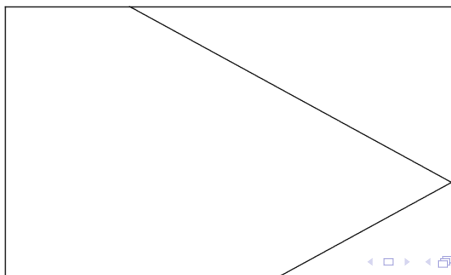
Instead, focus on one key idea:
“specular reflection”



Warm up problem:

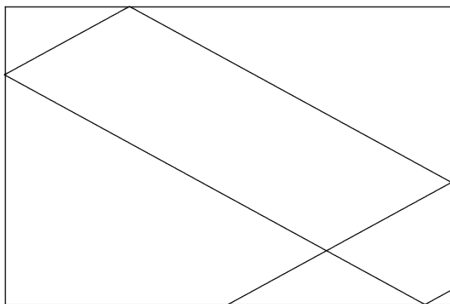
Draw the path of ONE billiard on a rectangular table through several collisions.

(1)



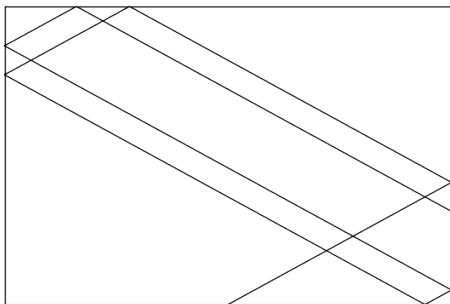
Warm up problem:

Draw the path of a billiard on a rectangular table through several collisions.



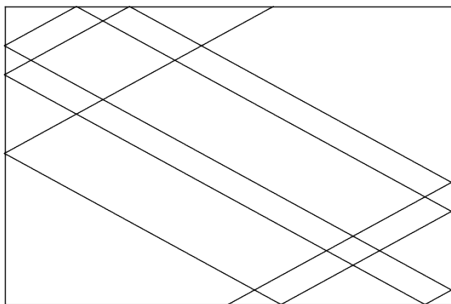
Warm up problem:

Draw the path of a billiard on a rectangular table through several collisions.



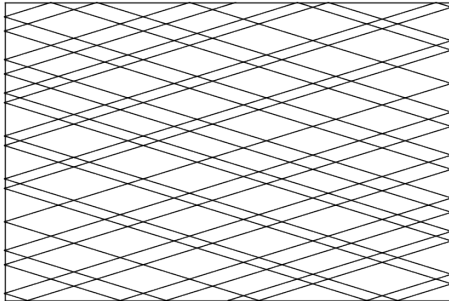
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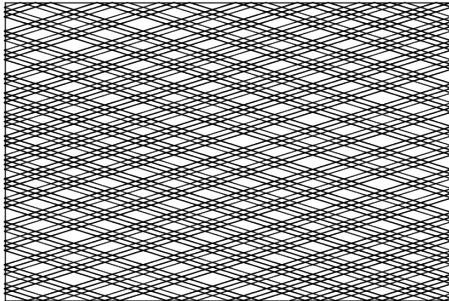
Rectangle billiards

One type of behavior: 41 collisions



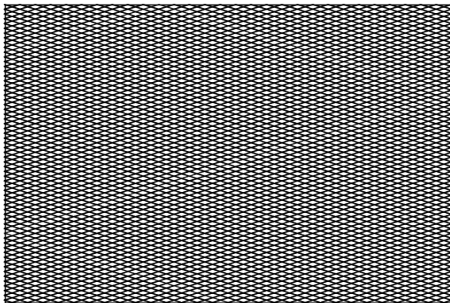
Rectangle billiards

One type of behavior: 221 collisions



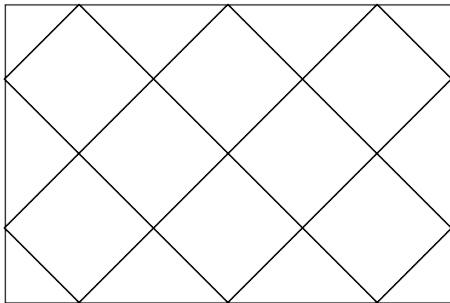
Rectangle billiards

One type of behavior: 362 collisions



Rectangle billiards

Another type: “periodic orbit” of period 10

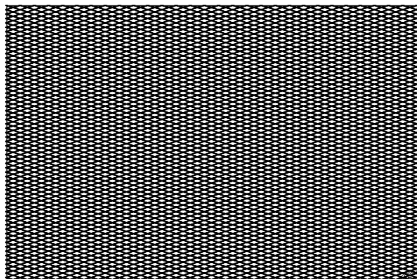
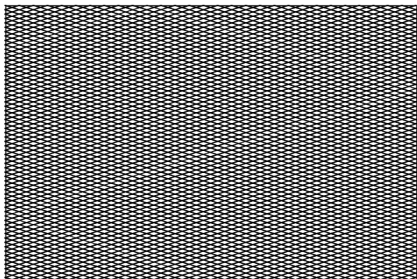


Next problem: some
more interesting
shapes!



Rectangle billiards

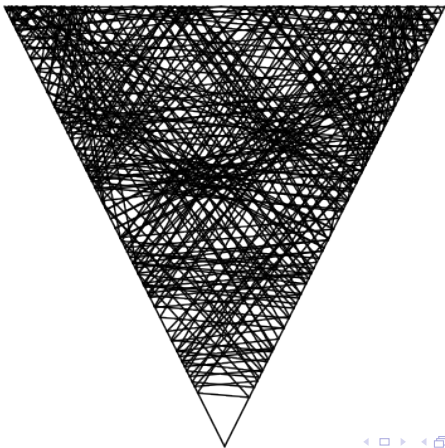
Actually the first one might be periodic too:



Next problem: some
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Triangle billiards



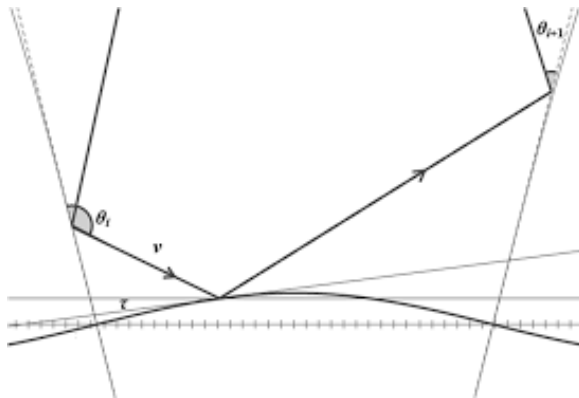
Things to notice:

- ▶ The model works for curved boundaries, not just lines
- ▶ This model assumes there is no friction, no loss of energy, and no spinning

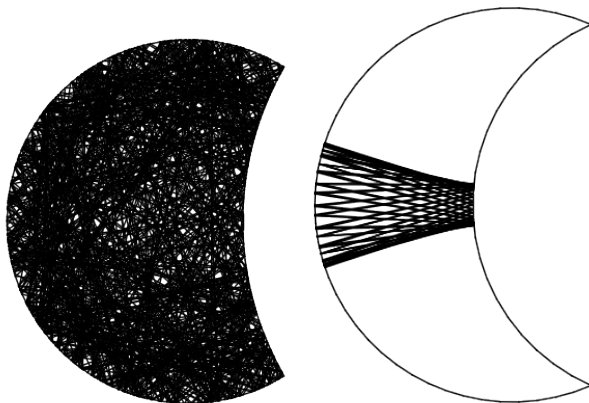


Specular collisions

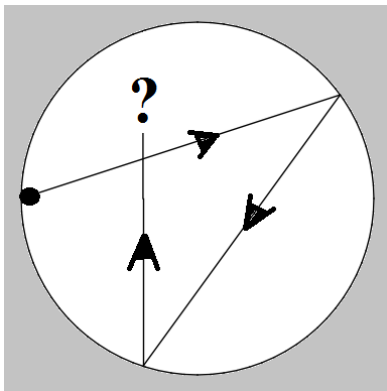
For curves, we use the tangent line:



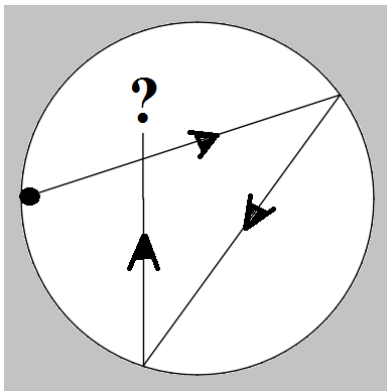
Moon billiards



Worksheet Question 4: What do billiards look like in a circular table?

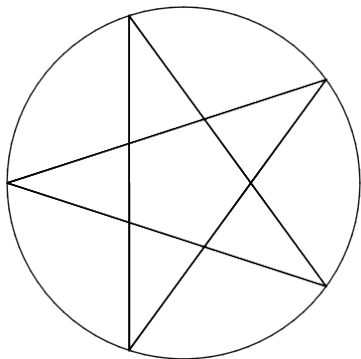


Worksheet Question 4: What do billiards look like in a circular table?



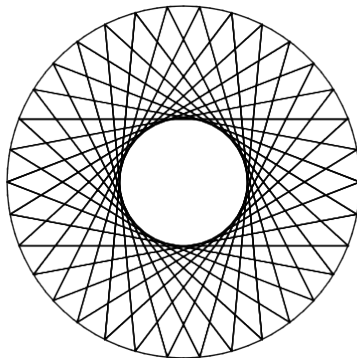
Question 1: What do billiards look like in a circular table?

That one looks like this:



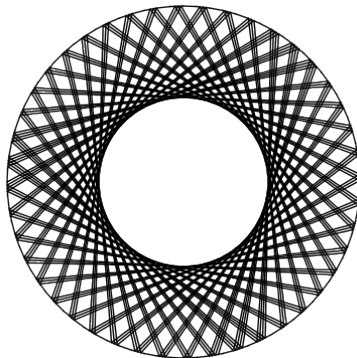
Question 1: What do billiards look like in a circular table?

With different direction:



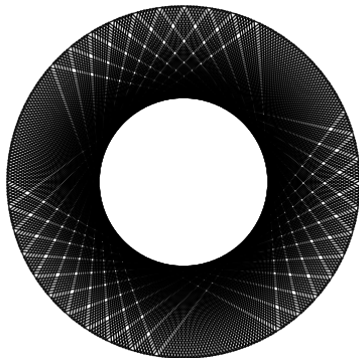
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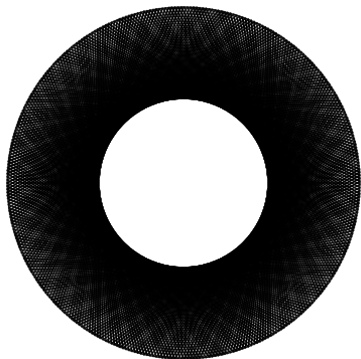
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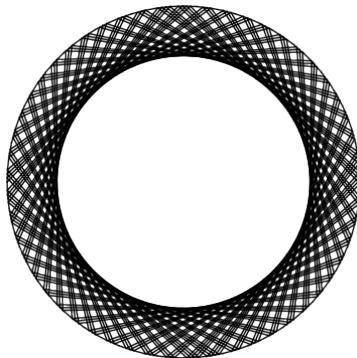
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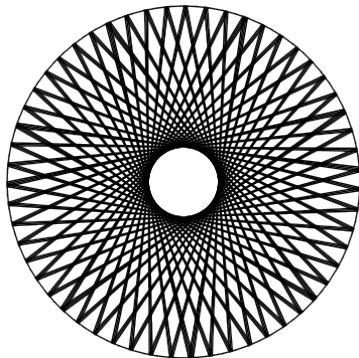
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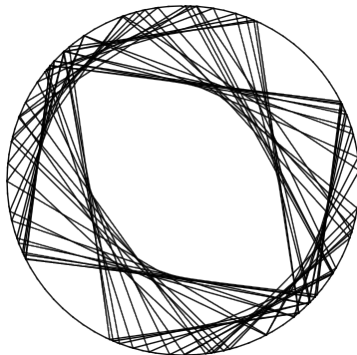


Question 1: What do billiards look like in a circular table?

With different direction:



Question 2: What is wrong with my circle?





Three types of behavior

- ▶ Periodic
- ▶ Nice but non-periodic
- ▶ Chaotic



Undergraduates Research Billiard Dynamics

 Fairfield University REU in Mathematics & Computational Science



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Projects 2016

This year's program runs from **May 30, 2016 - July 22, 2016.**



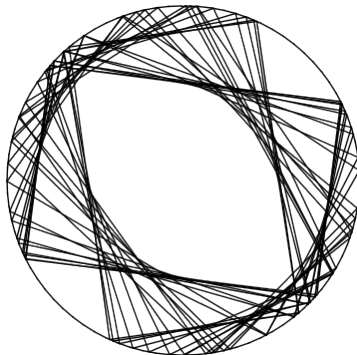
Yakov Sinai awarded the Abel Prize



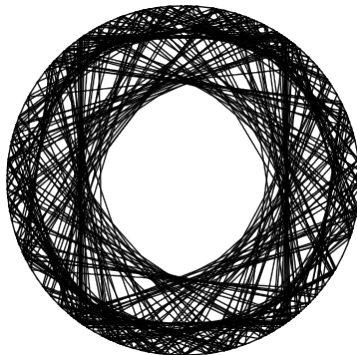
Can you make a periodic circle billiard?



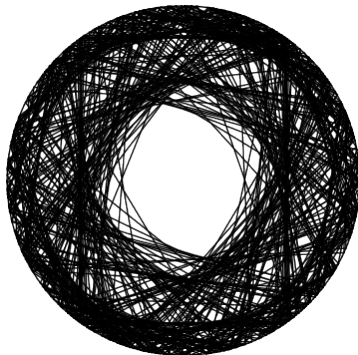
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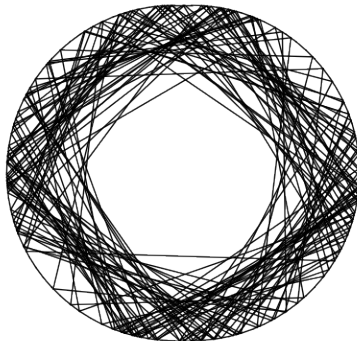
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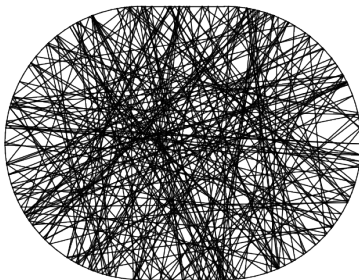
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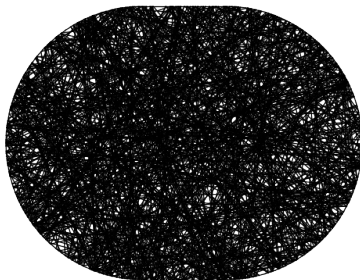
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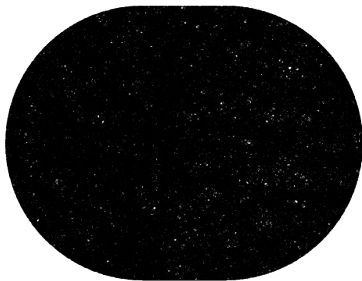
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Question: What is wrong with my circle?



Some really interesting tables

Worksheet question 6: ellipses and moons
and stadia and mushrooms



The mushroom billiard

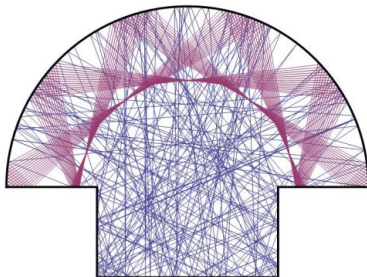


Figure by Carl Dettmann

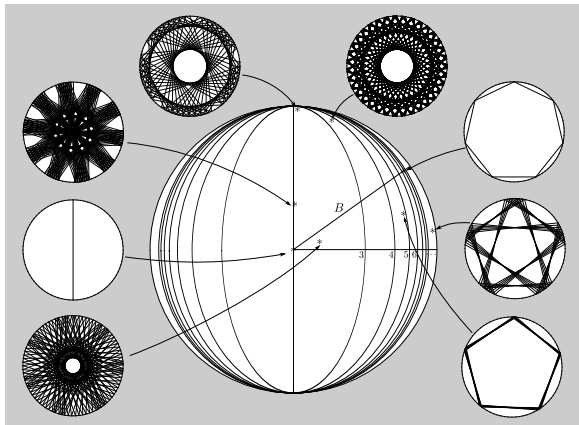


The ergodic game

How does our sampling rule change the “average”?



A new rule: “no-slip” circles



Applications of specular collisions

This simple model has many applications:

- ▶ Modeling fluids (Lorentz gases)



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Applications of specular collisions

This simple model has many applications:

- ▶ Modeling fluids (Lorentz gases)
- ▶ Brownian motion
- ▶ Heat transfer (How do things cool off?)
- ▶ Diffusion (How do mixtures spread out?)

