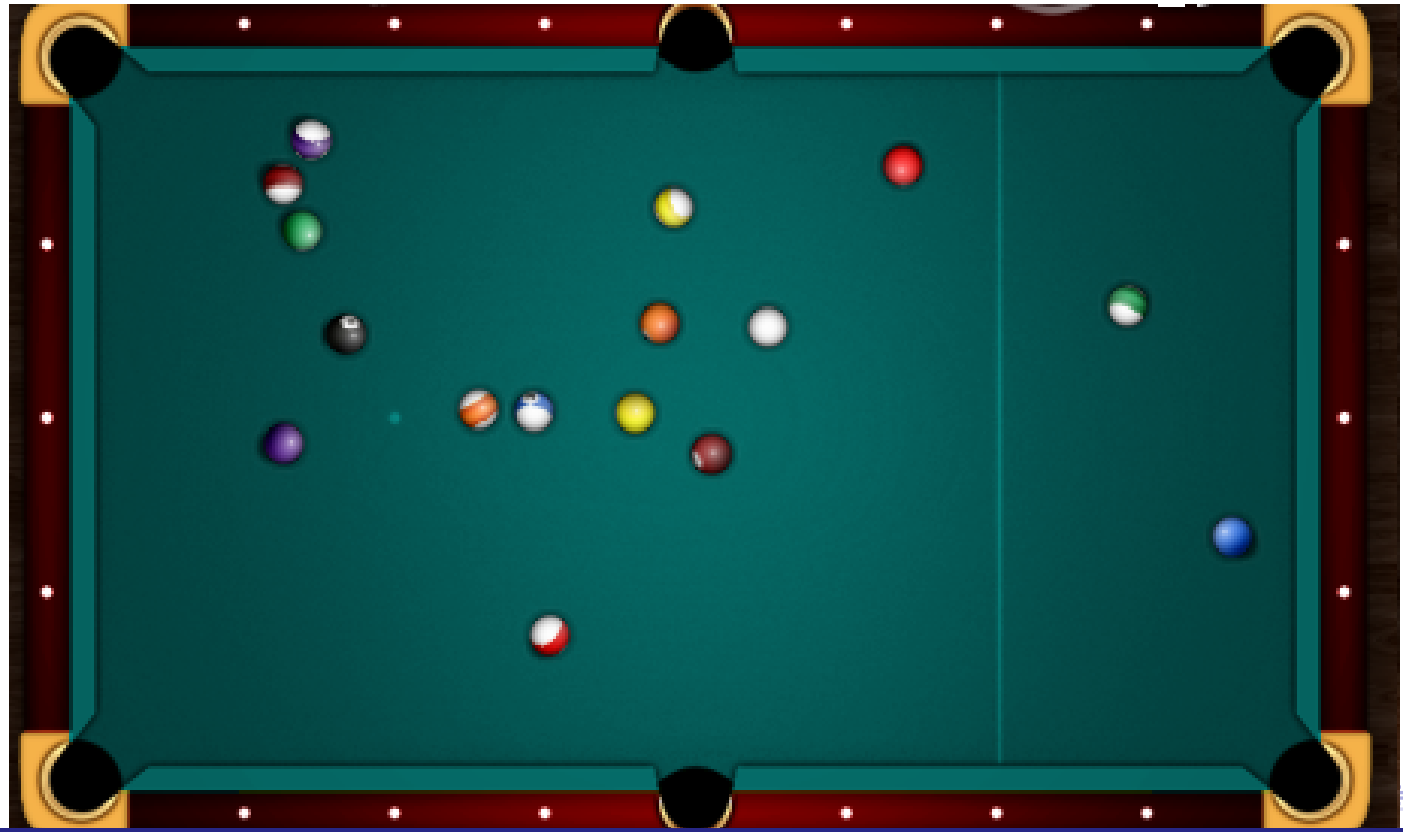
# The Mathematics of Billiards

## Washington University Math Circle

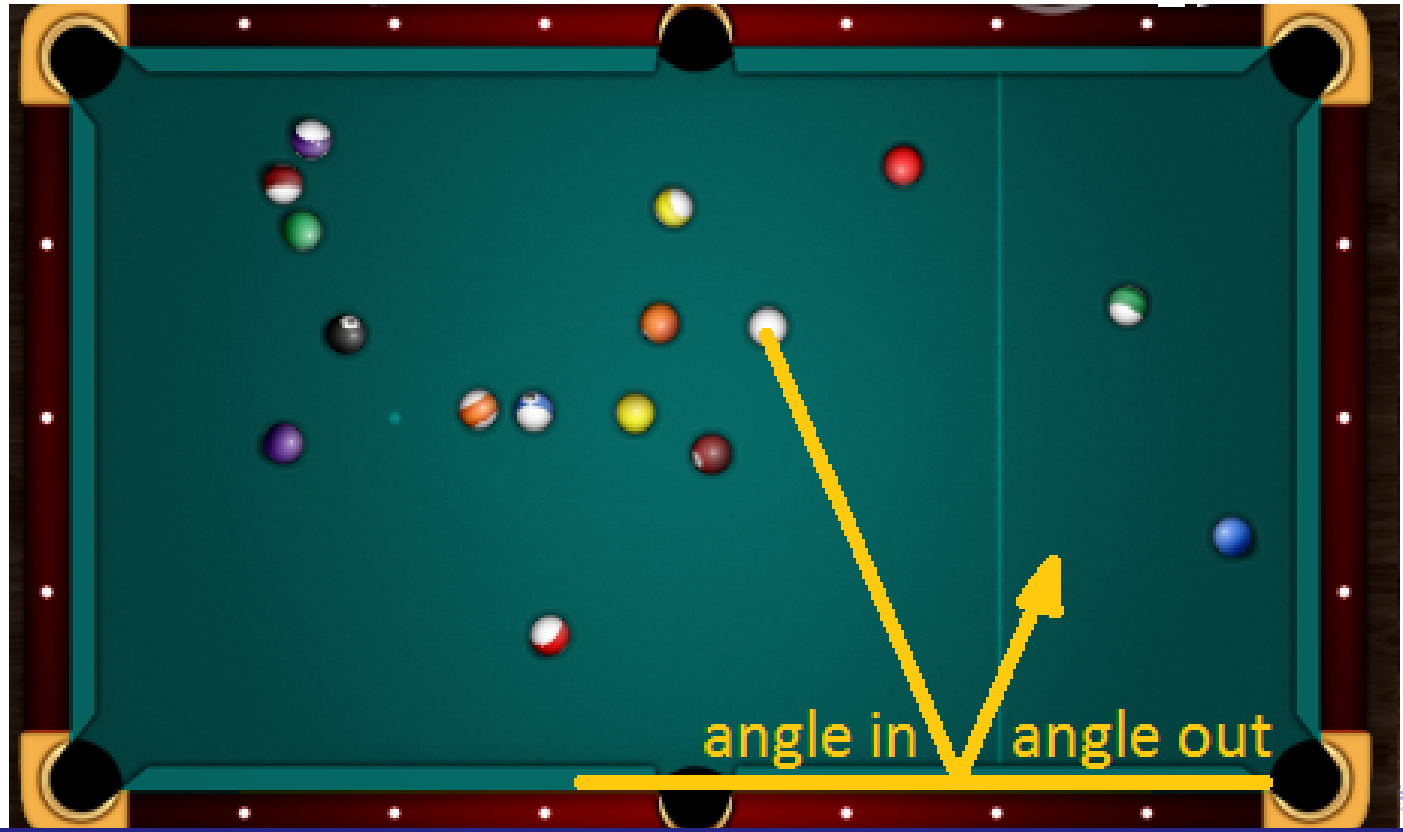
Chris Cox

March 6, 2016

# One thing you could do but we won't: play real billiards! ☹

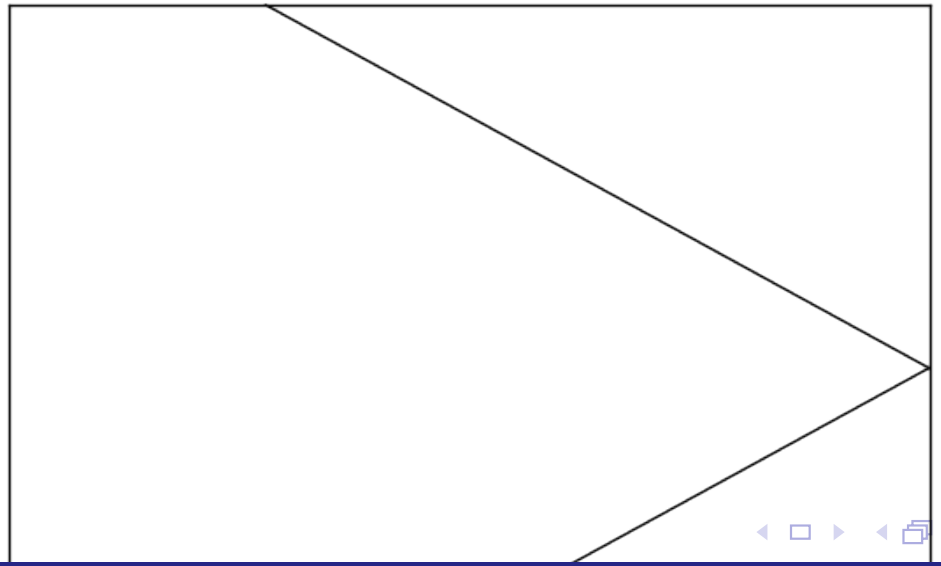


# Instead, focus on one key idea: "specular reflection"



# Warm up problem:

Draw the path of ONE billiard on a rectangular table through several collisions. (1)



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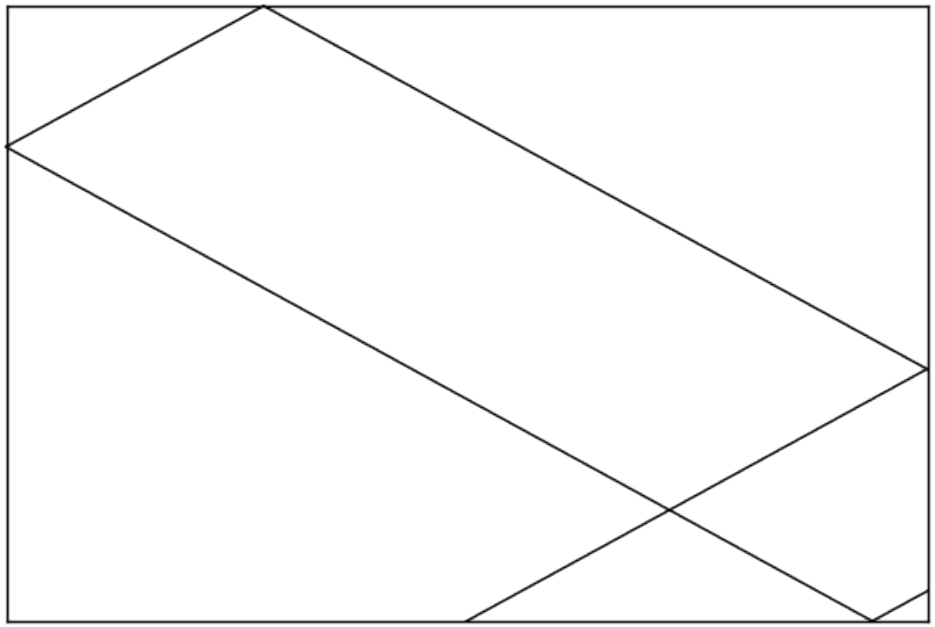

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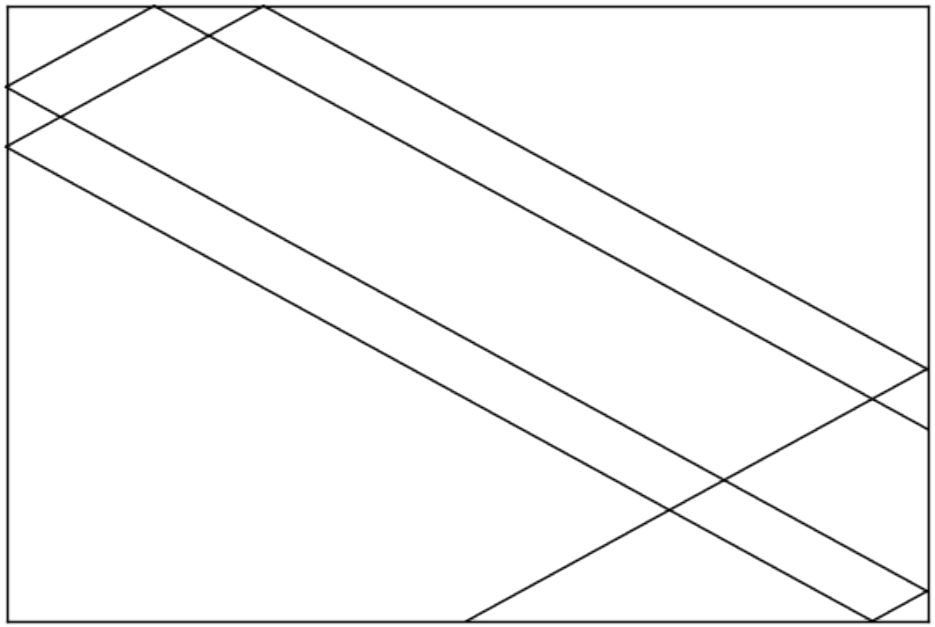

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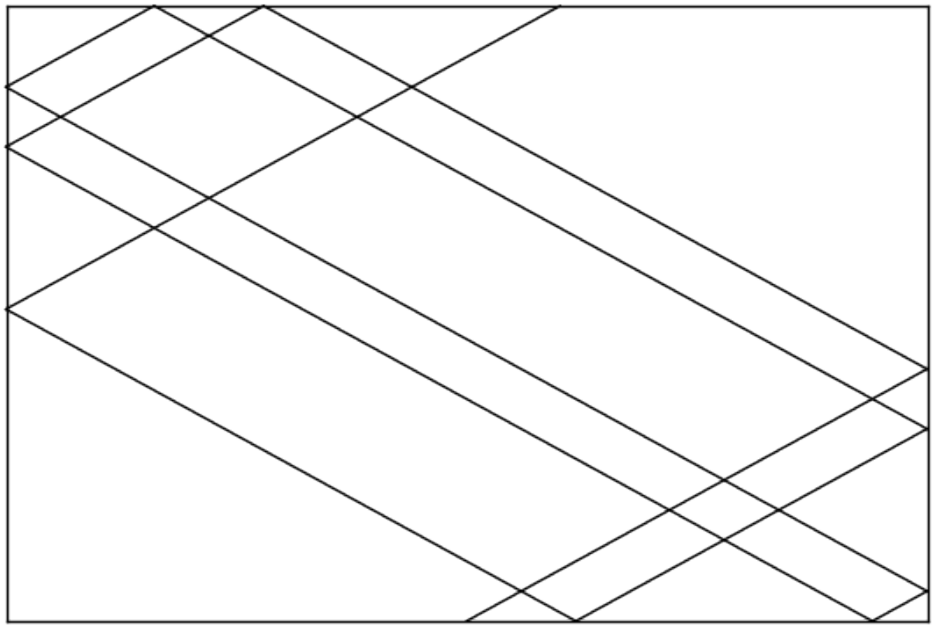
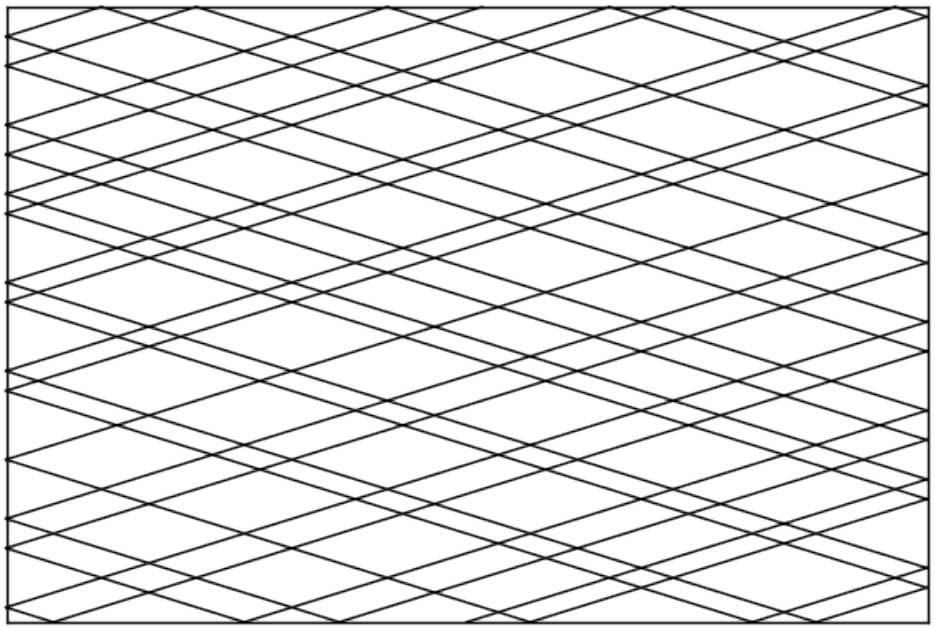


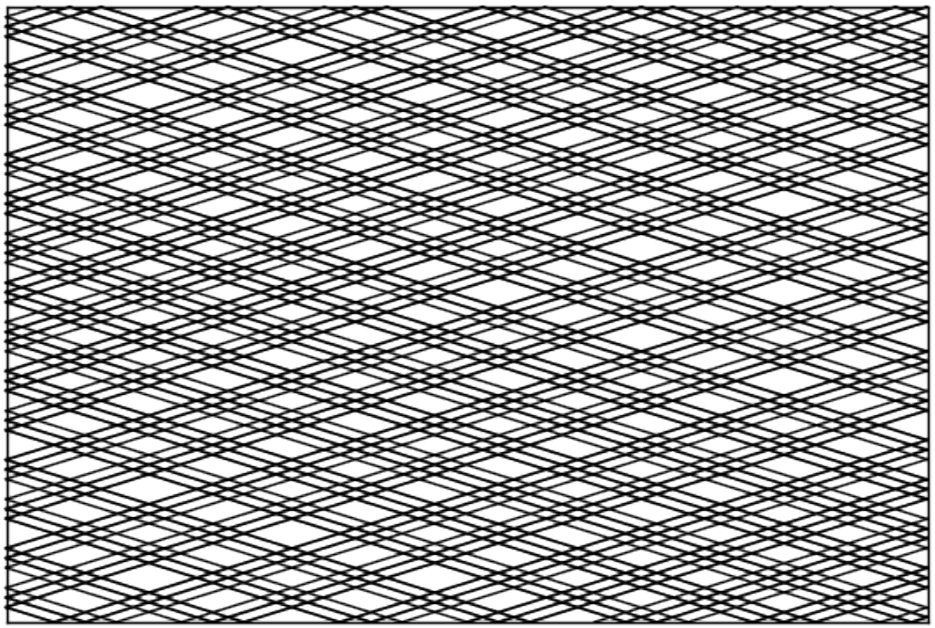
# Rectangle billiards

One type of behavior: 41 collisions



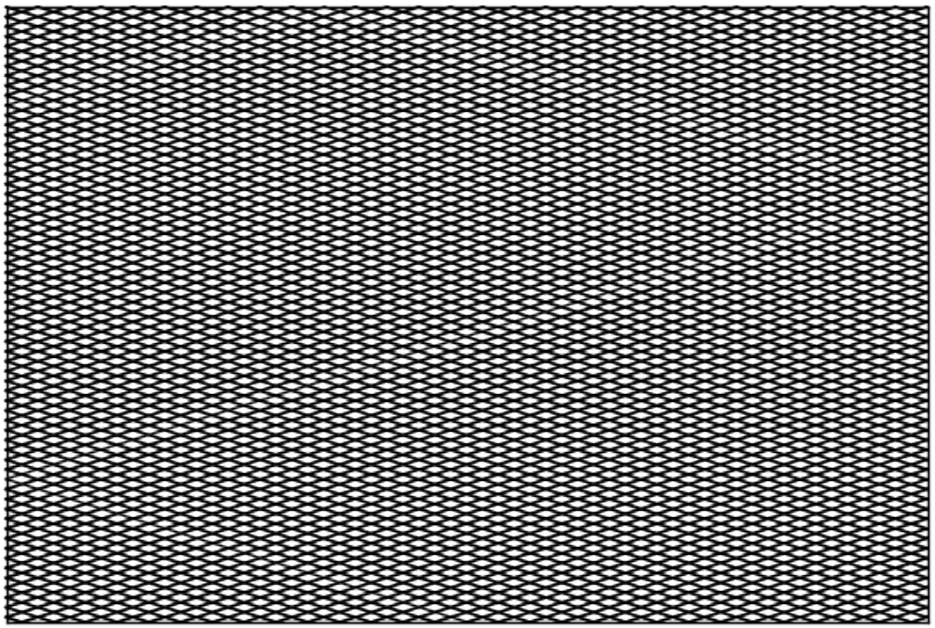
# Rectangle billiards

One type of behavior: 221 collisions



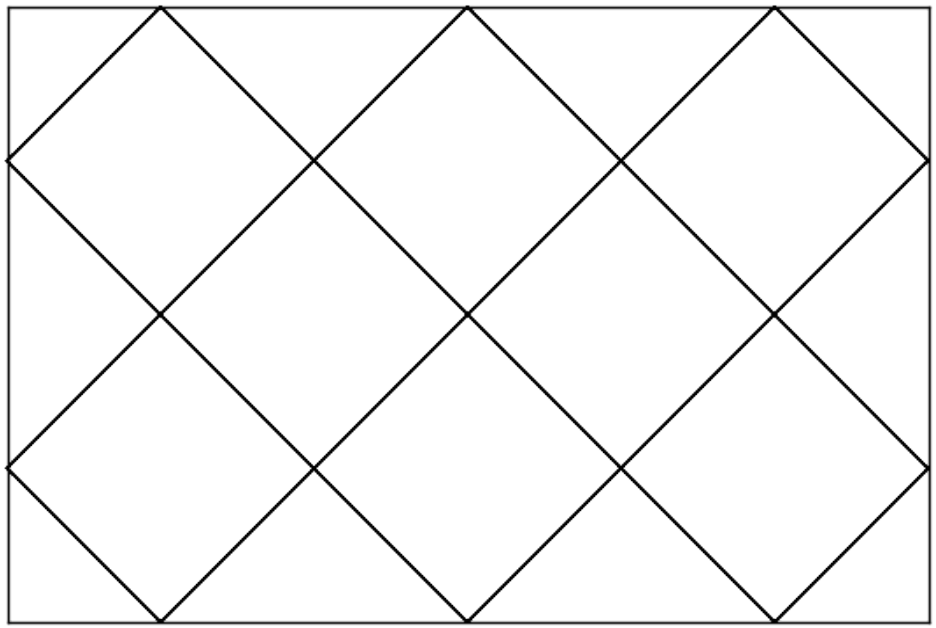
# Rectangle billiards

One type of behavior: 362 collisions



# Rectangle billiards

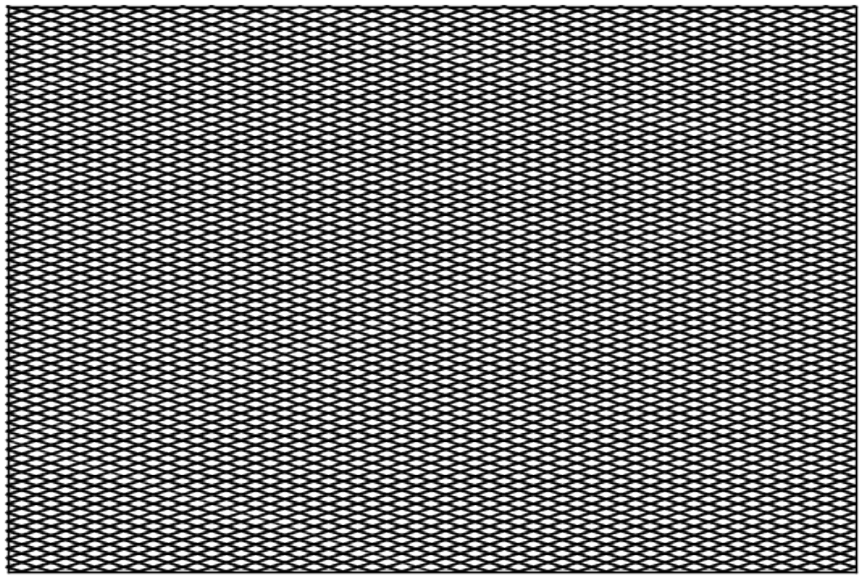
Another type: "periodic orbit" of period 10

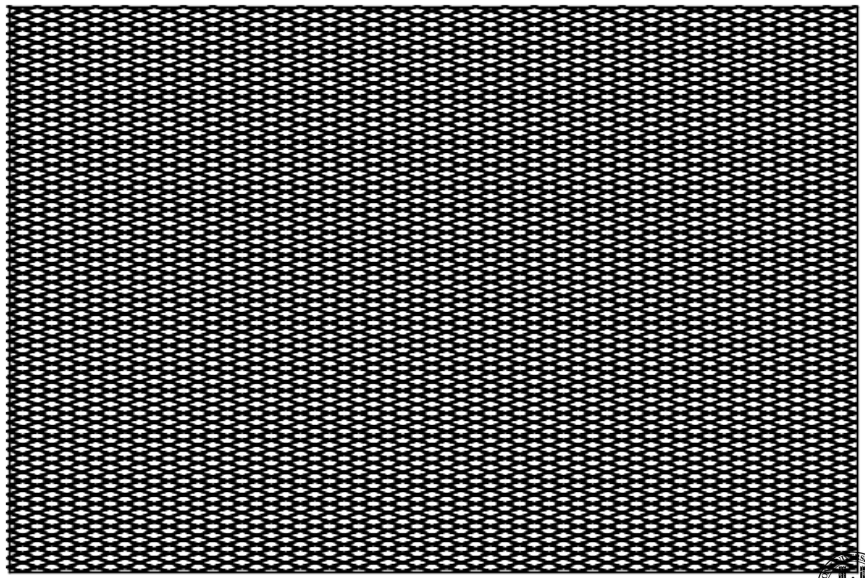


# Next problem: some more interesting shapes!

# Rectangle billiards

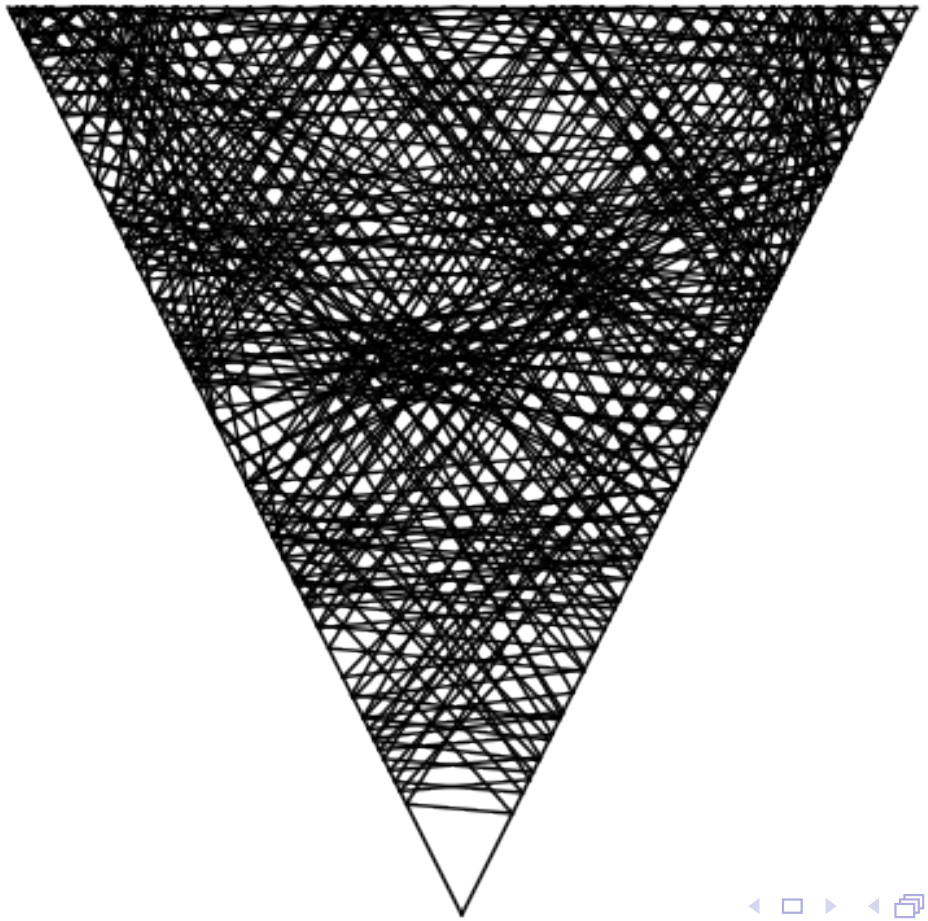
Actually the first one might be periodic too:





# Next problem: some more interesting shapes!

# Triangle billiards

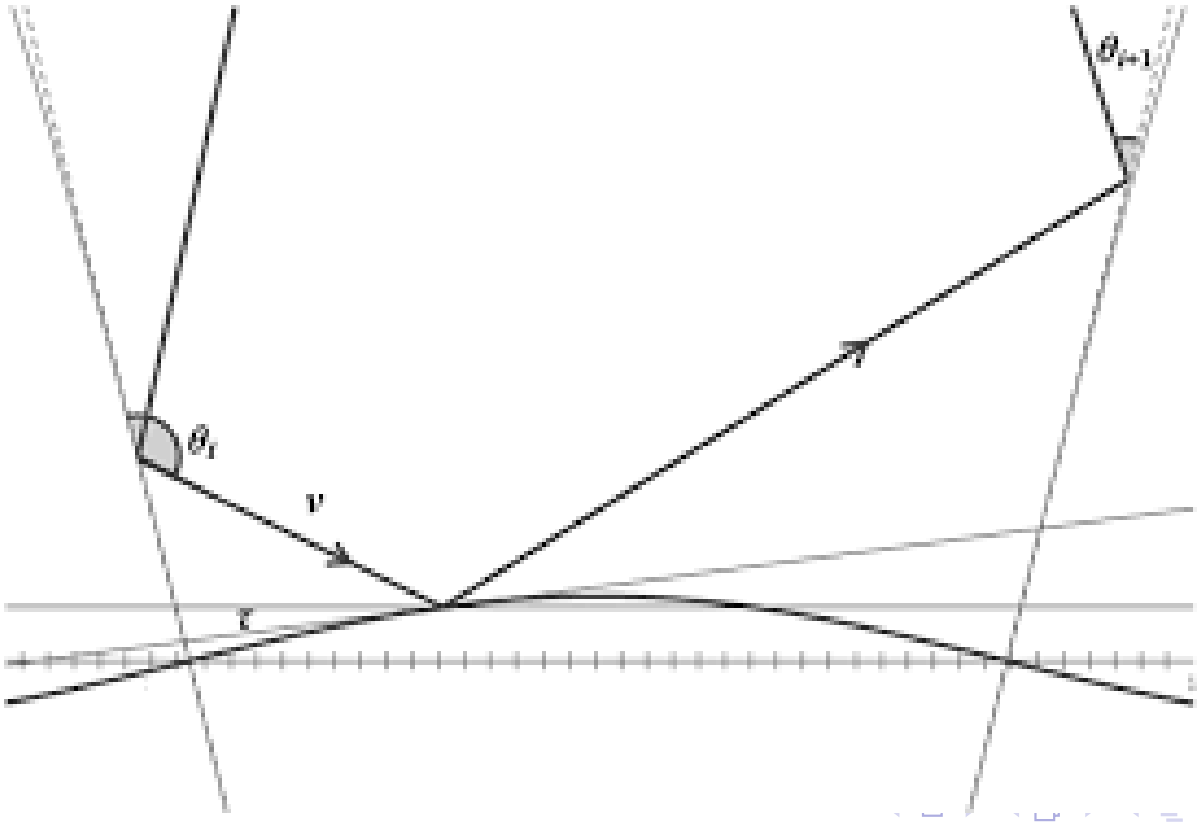


# Things to notice:

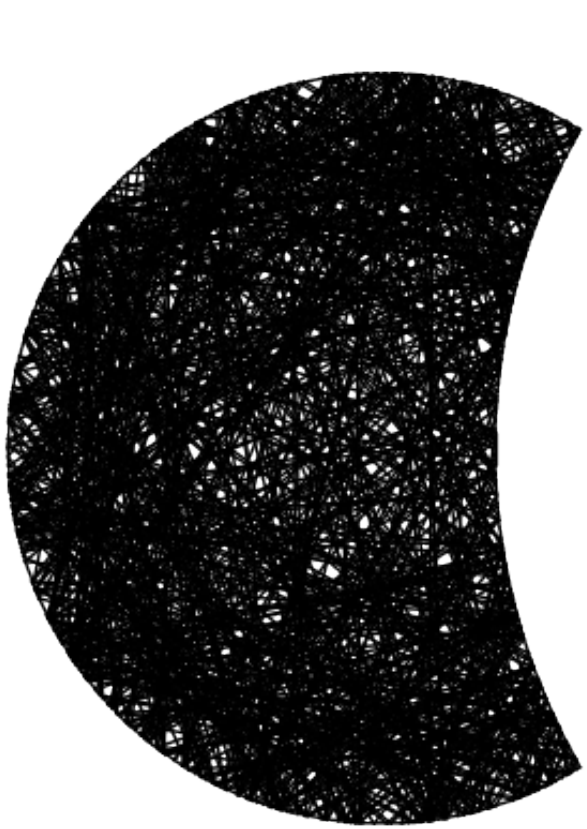
- The model works for curved boundaries, not just lines
- This model assumes there is no friction, no loss of energy, and no spinning

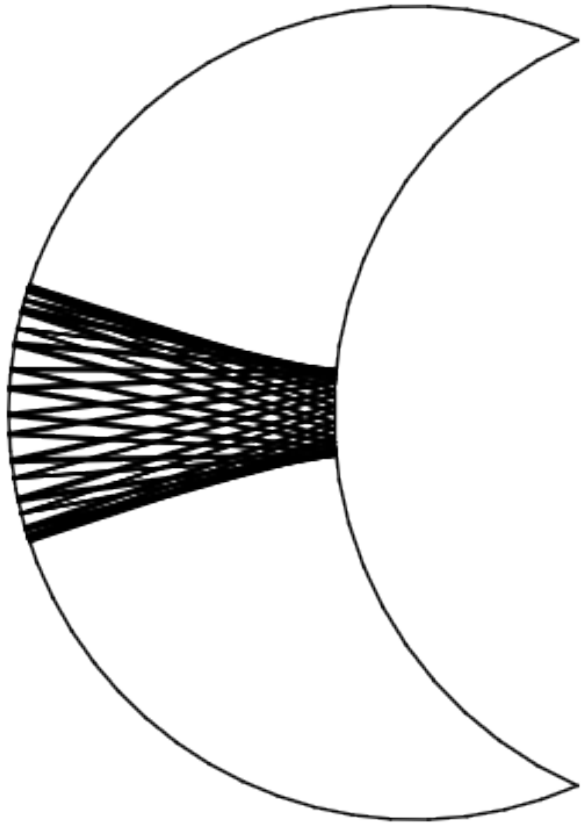
# Specular collisions

For curves, we use the tangent line:

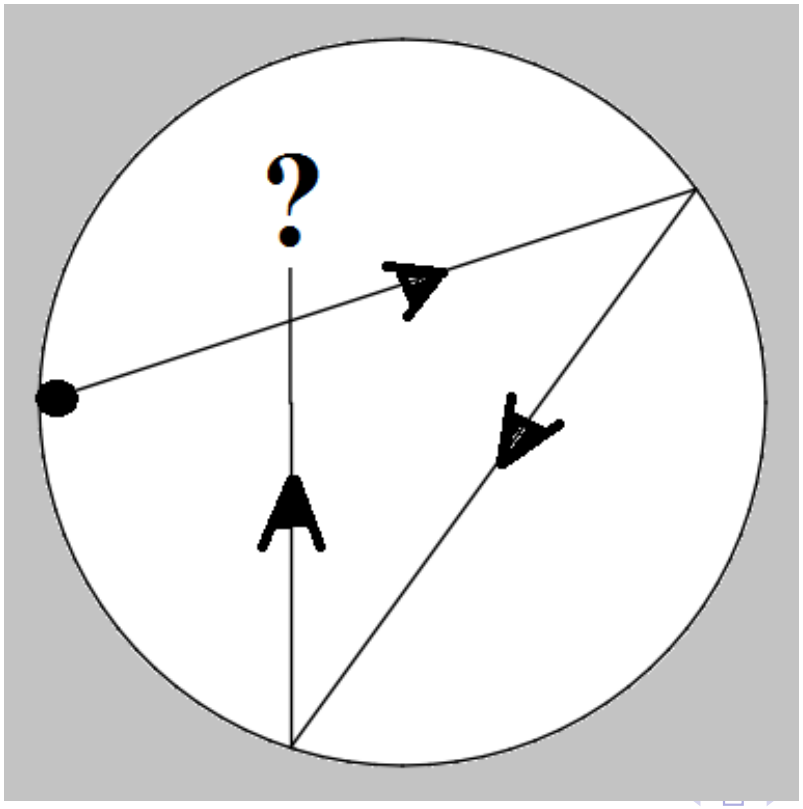


# Moon billiards





# Worksheet Question 4: What do billiards look like in a circular table?

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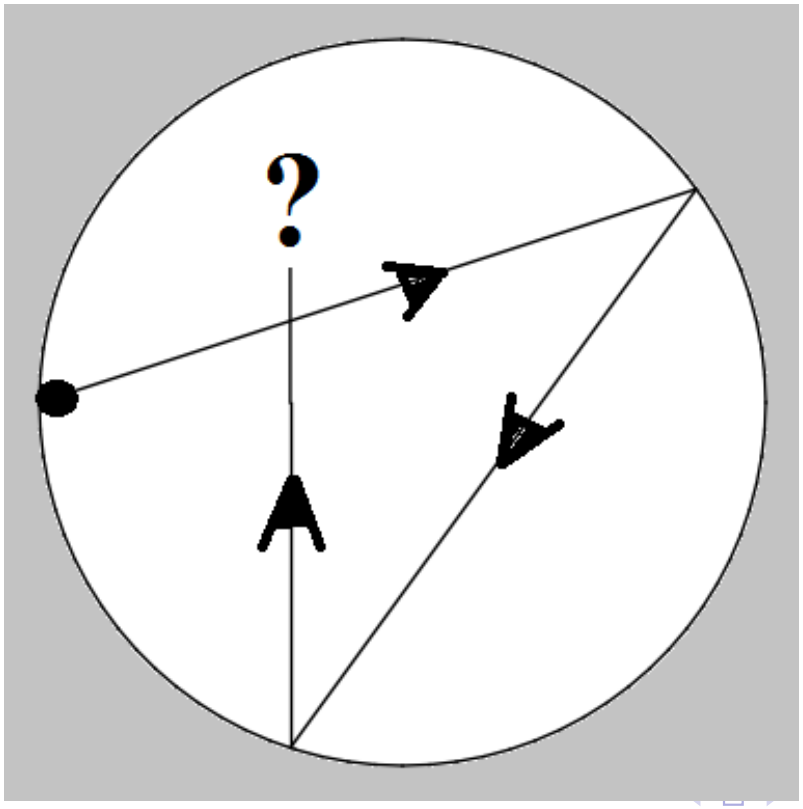
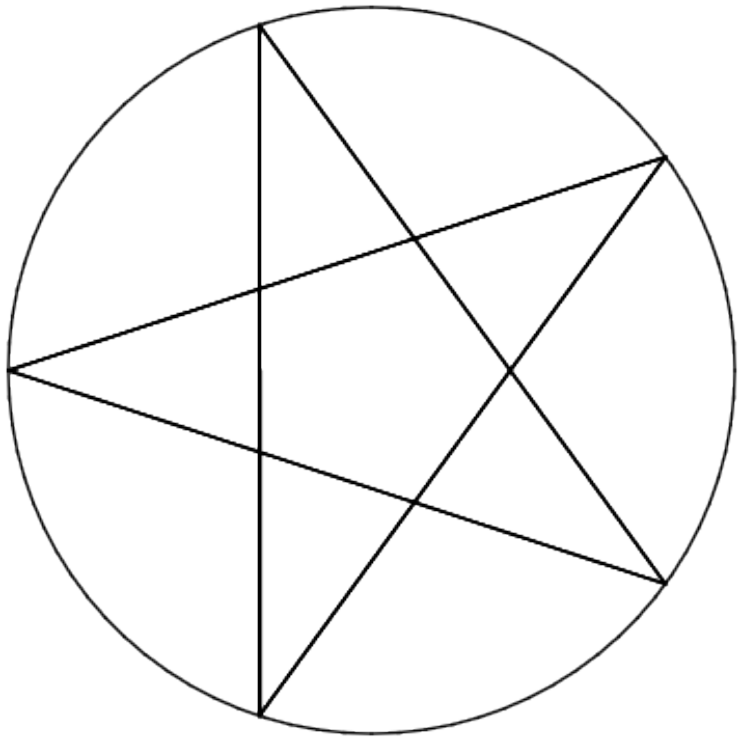



Question 1: What do billiards look like in a circular table?

# That one looks like this:



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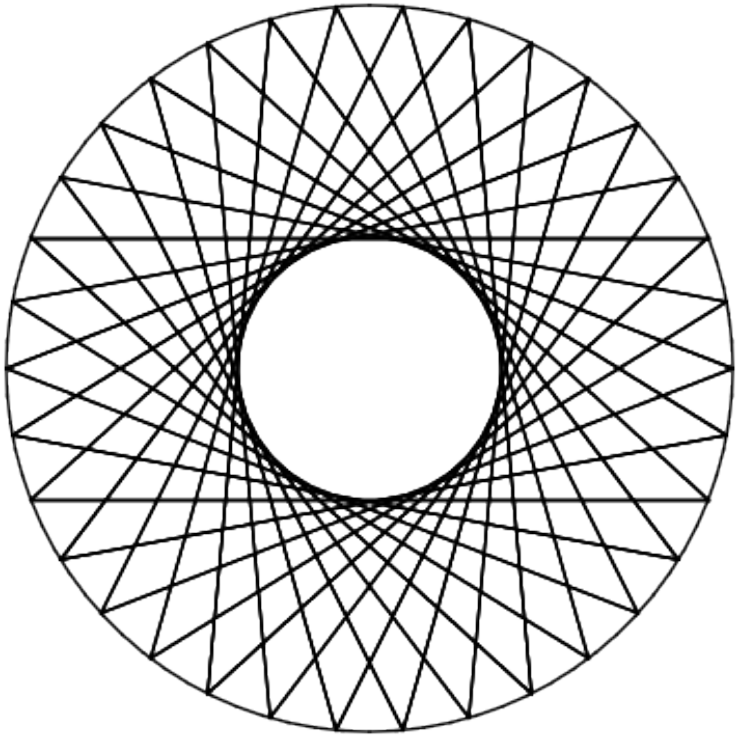

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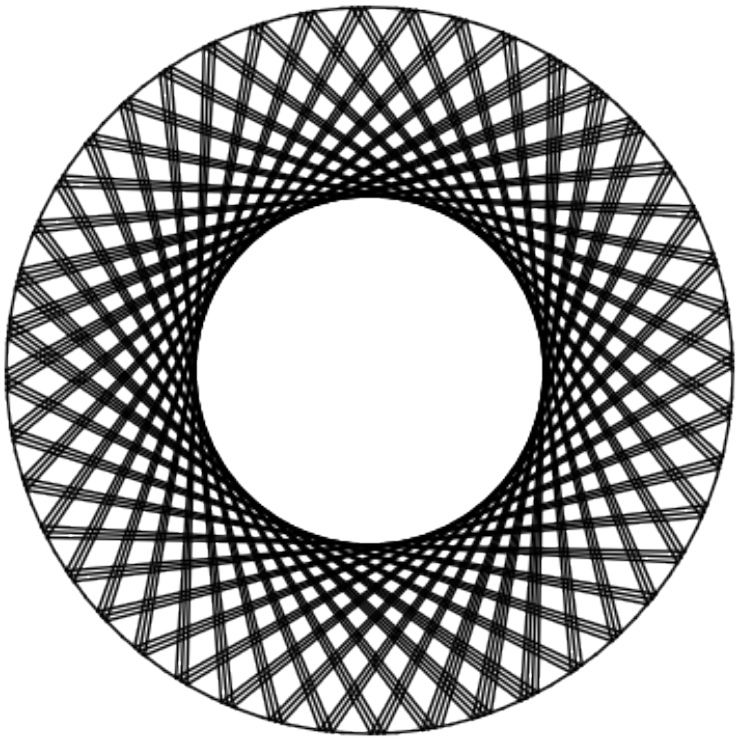

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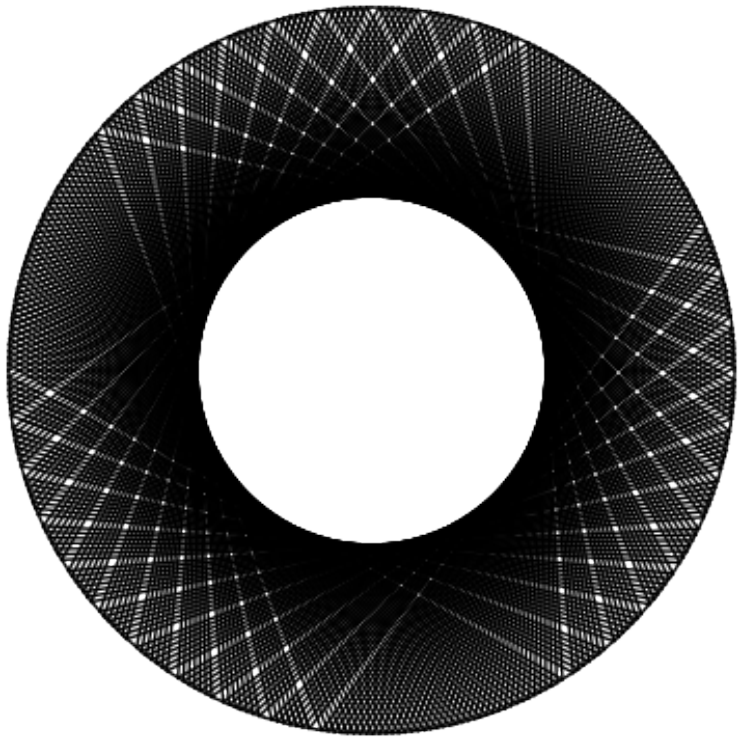

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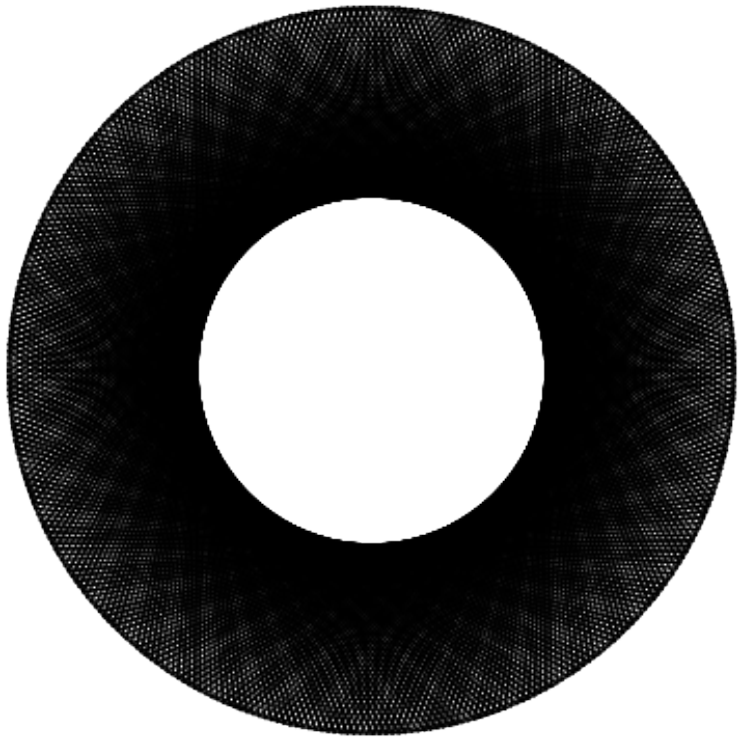

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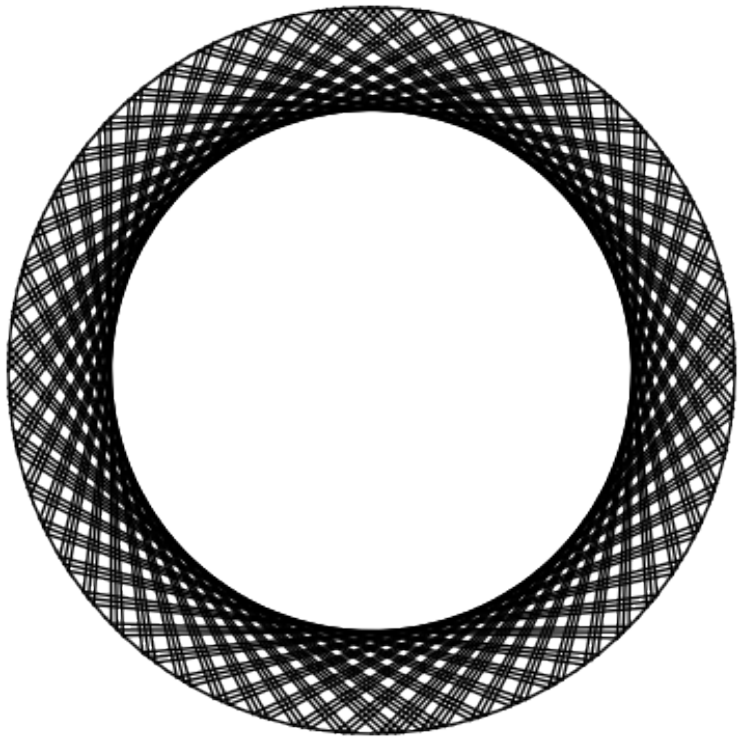

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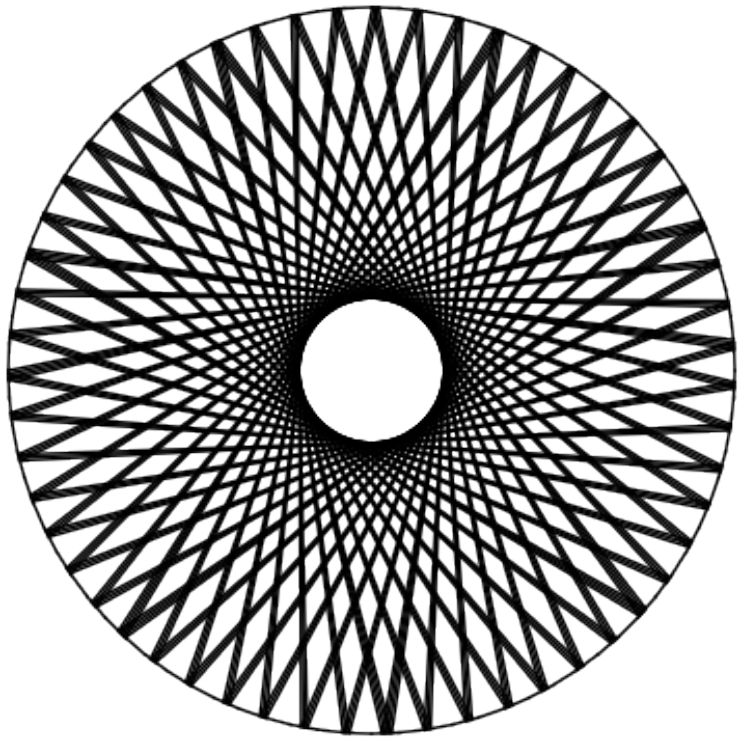
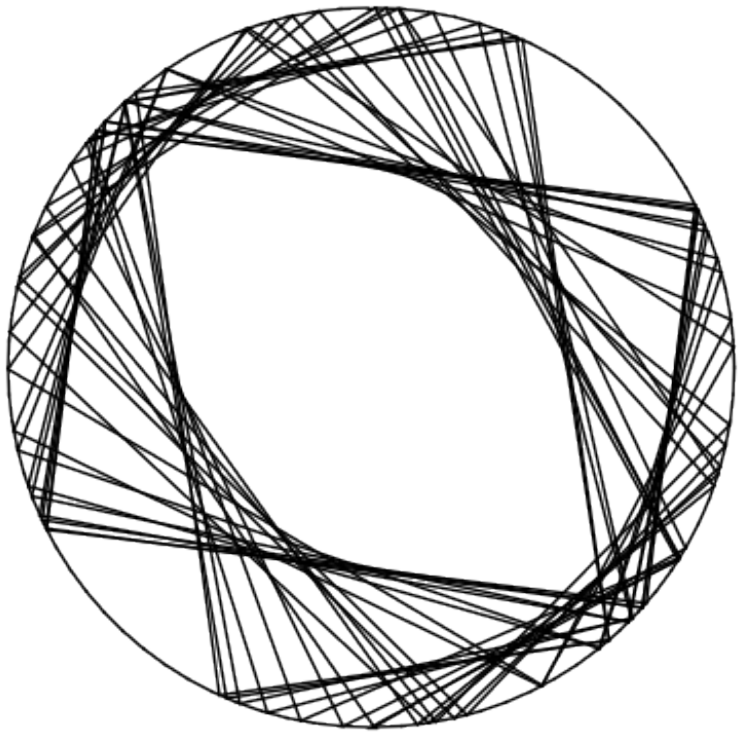


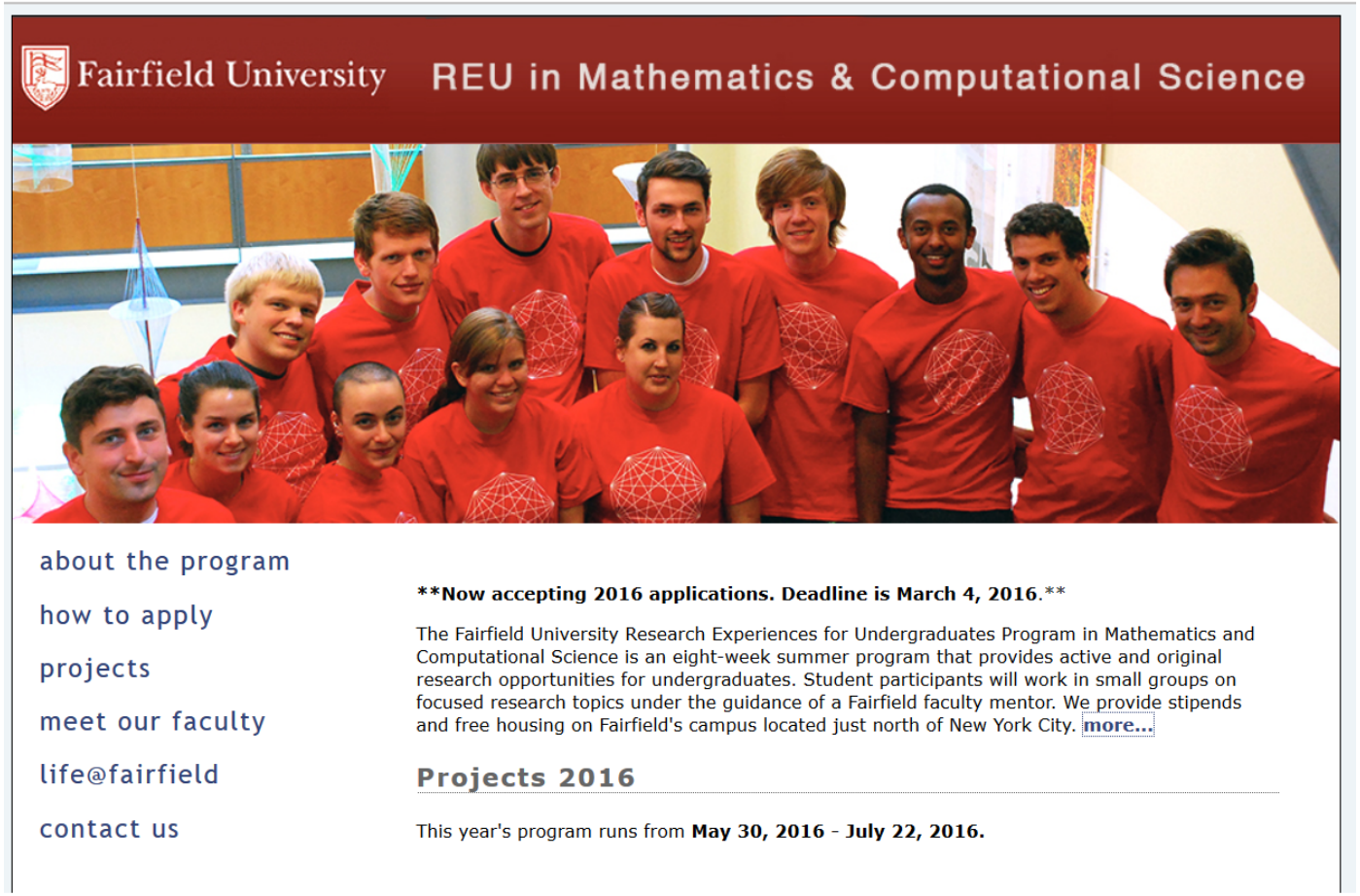
# Question 2: What is wrong with my circle?



# Three types of behavior

- Periodic
- Nice but non-periodic
- Chaotic

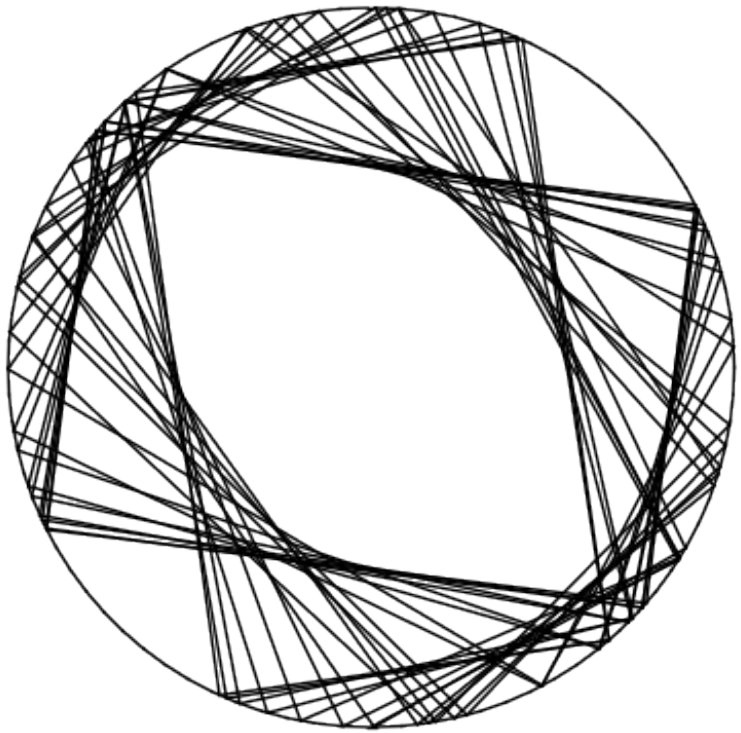
# Undergraduates Research Billiard Dynamics



# Yakov Sinai awarded the Abel Prize

# Can you make a periodic circle billiard?

# Question: What is wrong with my circle?



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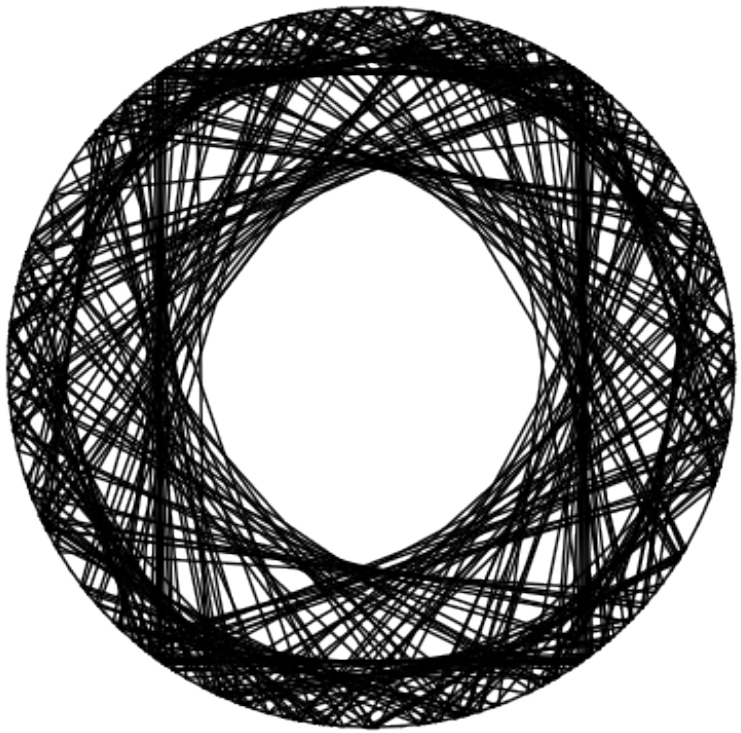

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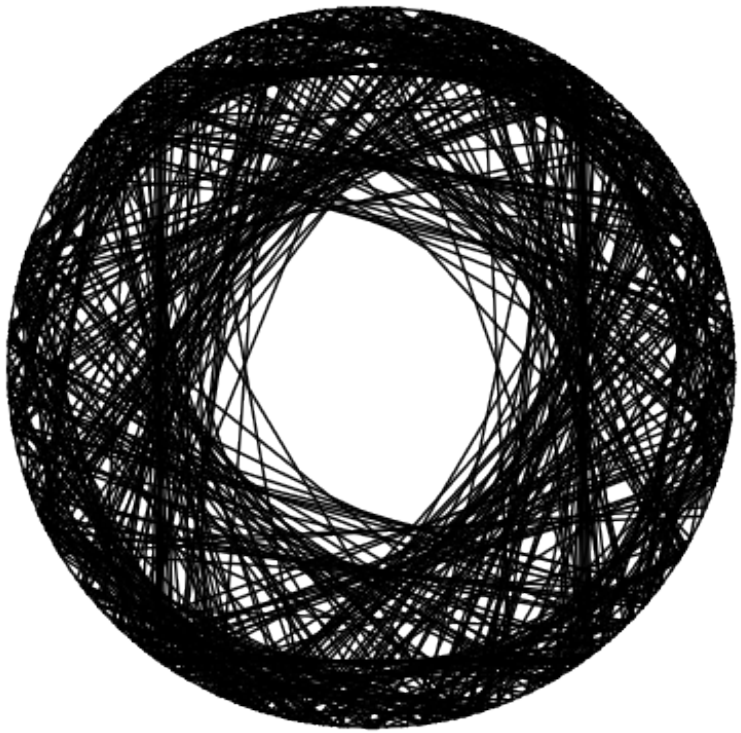

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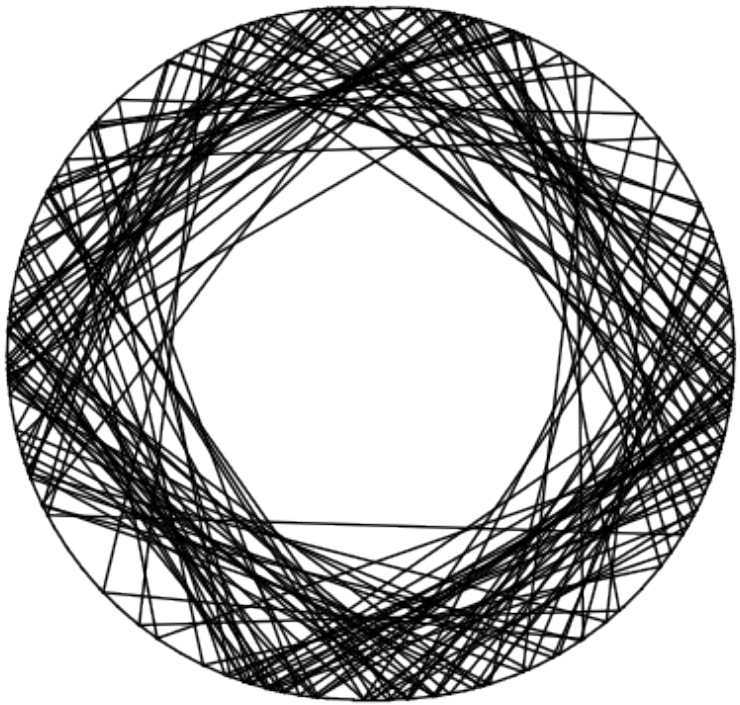

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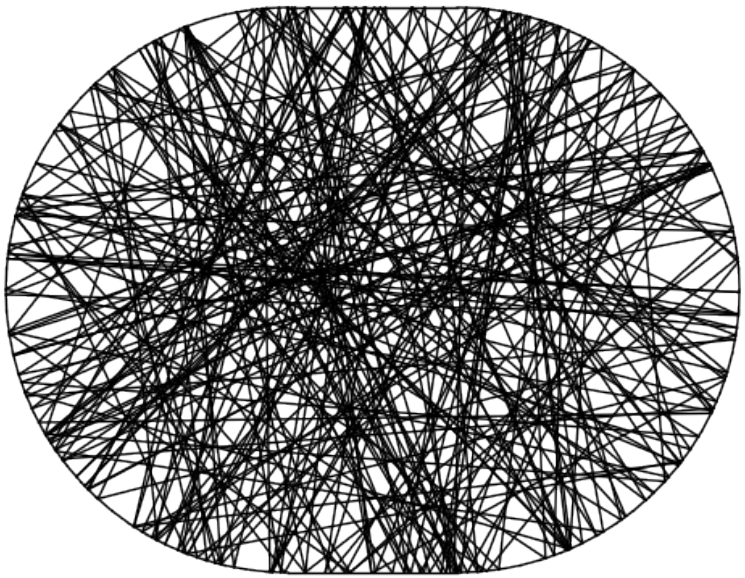

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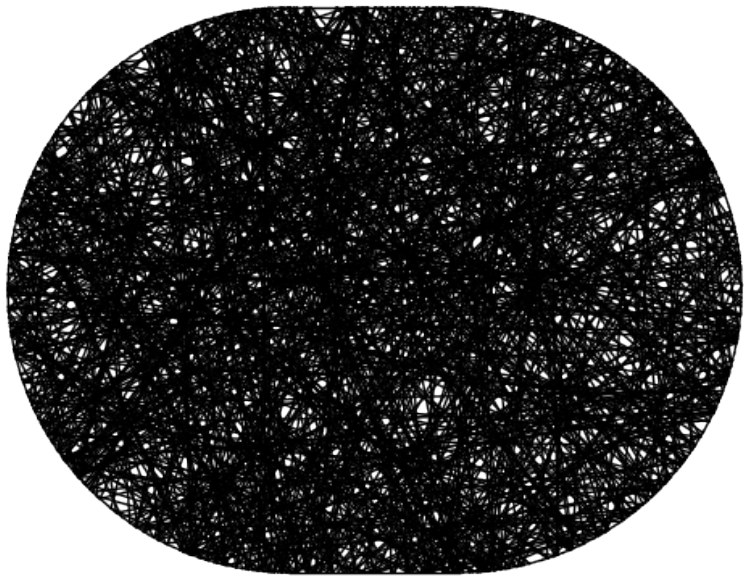

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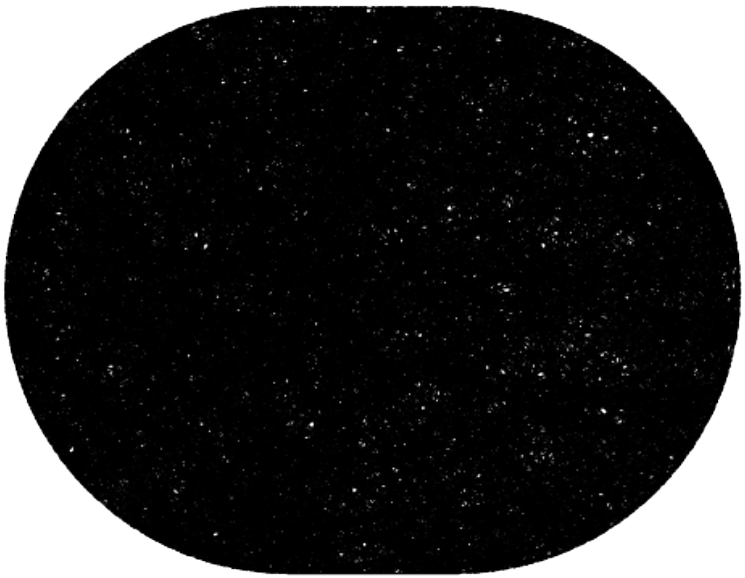


# Some really interesting tables

Worksheet question 6: ellipses and moons and stadia and mushrooms

# The mushroom billiard

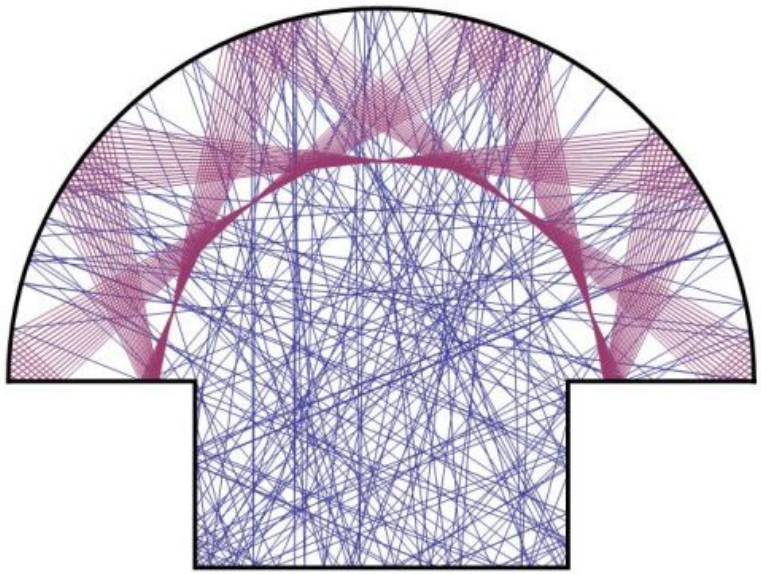
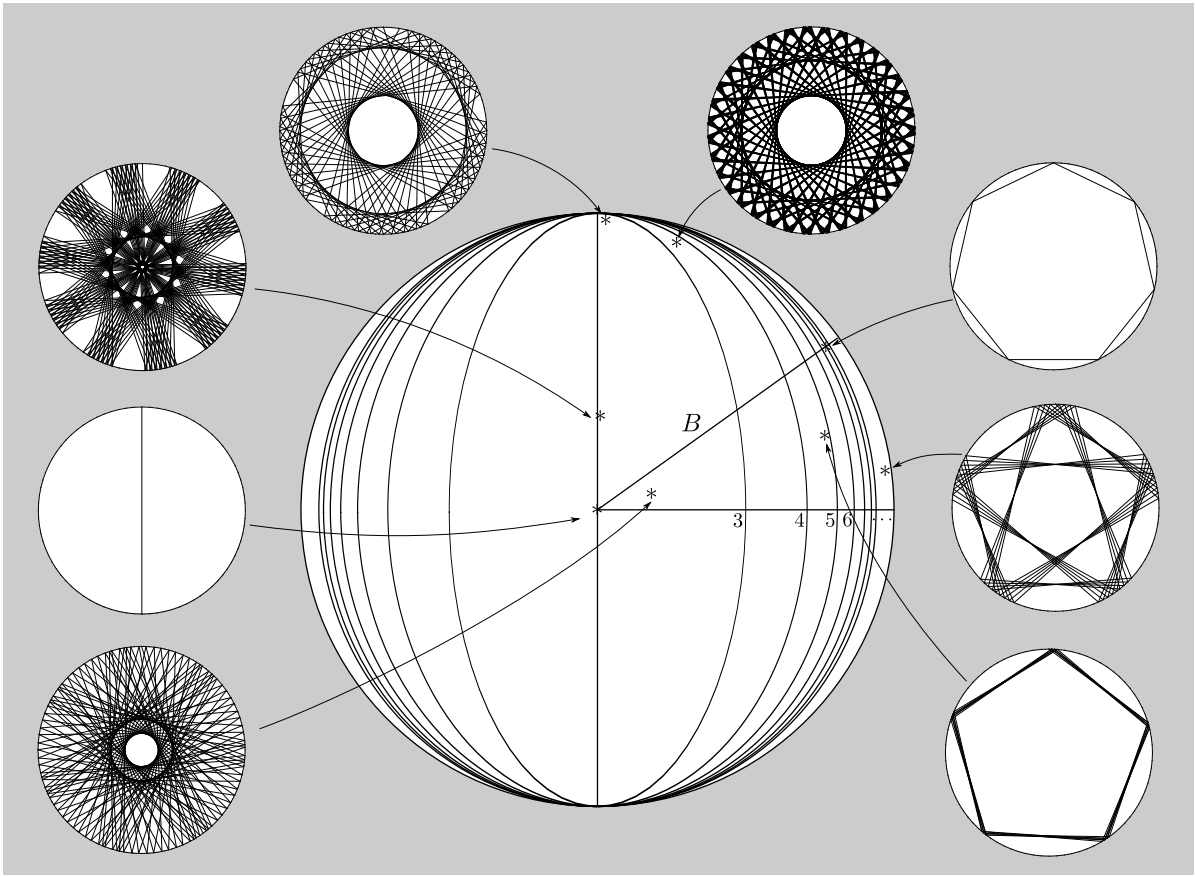


Figure by Carl Dettmann

# The ergodic game

How does our sampling rule change the "average"?

# A new rule: "no-slip" circles



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- Heat transfer (How do things cool off?)
- Diffusion (How do mixtures spread out?)