The Hidden Wonders of Numbers

PRESIDENTIAL AWARDS FOR EXCELLENCE IN MATHEMATICS AND SCIENCE TEACHING

Akehiko Takahashi

Topics for the Day

1. Who Am I? Mathematicians
2. Warm-up Questions*
3. Not in the Math Textbooks
4. Topology Genus*
5. How do you make them equal?*
6. 1 = .9999…
7. Vampire numbers (Clifford Pickover)
8. Prime numbers…the crown jewel of number theory
9. Goldbach Conjecture, Vinogradov Theorem
10. The Sieve of Eratosthenes and Yitang Zhang
11. Luhn Algorithm
12. Collatz Conjecture and hailstone numbers
13. Conway Sequence giving rise to the Game of Life*
14. Foxtrot and Perrin Sequence*
15. Moser's Circle Problem 1, 2, 4, 8, 16, ?*
16. Factorials, subfactorial, factorion, superfactorial
17. Lazy Caterer's Sequence, Floyd Triangle, etc*
18. Digital Roots (Henry Dudeney)*
19. Nobu's Ladder*
20. Benford's First Digit Law
21. Infinite Tower of Powers, Tetration
22. Grandi's Sequence
23. Golomb Ruler*
24. Vedic Math*
25. Sabermetrics (B. James), Mediant Fractions, Ford Circles
26. Sicherman's Difference Triangle*
27. The devil with a monkey wrench in the number game*
28. 5 challenging problems for the gifted in math*


a
b
c
NORGE
550
1802-1829
d
e

Warm-up Questions

1. $\lfloor 2.1 \rfloor =$
2. $\lfloor 2.7 \rfloor =$
3. $\lfloor -2.7 \rfloor =$
4. $\lceil 2.1 \rceil =$
5. $\lceil 2.7 \rceil =$
6. $\lceil -2.1 \rceil =$
7. Which is heavier one oz of gold or one oz of iron ore?
8. Which is heavier one pound of gold or one pound of iron ore?
9. What is the sum of the interior angles of any triangle? Is it always 180° ?
10. BAD BABE, DEAD, FEED FACE, BOOB1E5, CAD, Are they all legitimate numbers?
11. Which is greater: .9999999999999999... or 1?

Some Hidden Wonders of Numbers

You already know 4 ! = 4 x 3 x 2 x 1= 24

But what about the following?

1) !4
2) 9! !
3) 8 #
4) sf(4)
5) de Montmort number
6) $^{2}5$
7) x↑↑∞
8) (ϕ, θ, ψ)
9) name of 3 coordinates system (plane not 3D x, y, z)

10) Name the theorem for the sum of the distance from *any* interior point to the sides of an equilateral triangle equals the length of the triangle's altitude.

11) 1 – 1 + 1 – 1 + 1 – 1 + 1 – 1 + 1 – 1… =

12. Fujita Scale

8809 = 6

7111 = 0

2172 = 0

6666 = 4

1111 = 0

3213 = 0

7662 = 2

9313 = 1

0000 = 4

2222 = 0

3333 = 0

5555 = 0

8193 = 3

8096 = 5

1012 = 1

7777 = 0

9999 = 4

7756 = 1

6855 = 3

9881 = 5

5531 = 0

2581 = ?

How do you make them equal without changing digits?

100 = 102

103 = 112

#10 = #7

560 = 600

0 = 1

10 + 4 = 2

1 + 1 = 10

1 + 1 = 0

1 + 1 = 1

Supply the missing number in the following sequence:

10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24, ___, 100, 121, 10000

Which is greater?

.9999999... ____ 1

Method 1

1/9 = .1111111111111111...

2/9 = .2222222222222222...

3/9 = .3333333333333333...

8/9 = .8888888888888888...

9/9 = .9999999999999999...

Method 2

x = .99999... then10x = 9.99999...

10x - x = 9.99999... - .99999...= 9

9x = 9 x = 1 then .99999... = 1

Method 3

.99999999...= .9 + .09 + .009 + .0009...

sum of infinite geometric series s = a/(1 - r)

a = .9 r = .1 therefore .9/(1 -.1) = .9/.9 = 1

Vampires Numbers (Clifford Pickover)

there are many vampire numbers, but there are only 7 four-digits vampire numbers

$1260 = 21 \cdot 60$ $\quad$ $1395 = 15 \cdot 93$

$1435 = 35 \cdot 41$ $\quad$ $1530 = 30 \cdot 51$

$1827 = 21 \cdot 87$ $\quad$ $2187 = 27 \cdot 81$

Find the 7^{th} vampire number and its fangs

Hint: all digits are even with the last digit 0 with R = 4
Genus 6 with fangs with equal genus number

An important theoretical result found:Modulo 9 or the remainder of 4 (this number signifies death in Japanese)
If x y is a vampire number then $xy = x + y \pmod 9$

Prime Numbers

(p, 2p+1) p = Sophie Germain prime

(p, p+2) Twin primes

(p, p+4) Cousin primes

(p, p+6) Sexy primes

$M_p = 2^p - 1$ Mersenne prime

the Great Internet Mersenne Prime Search (GIMPS).

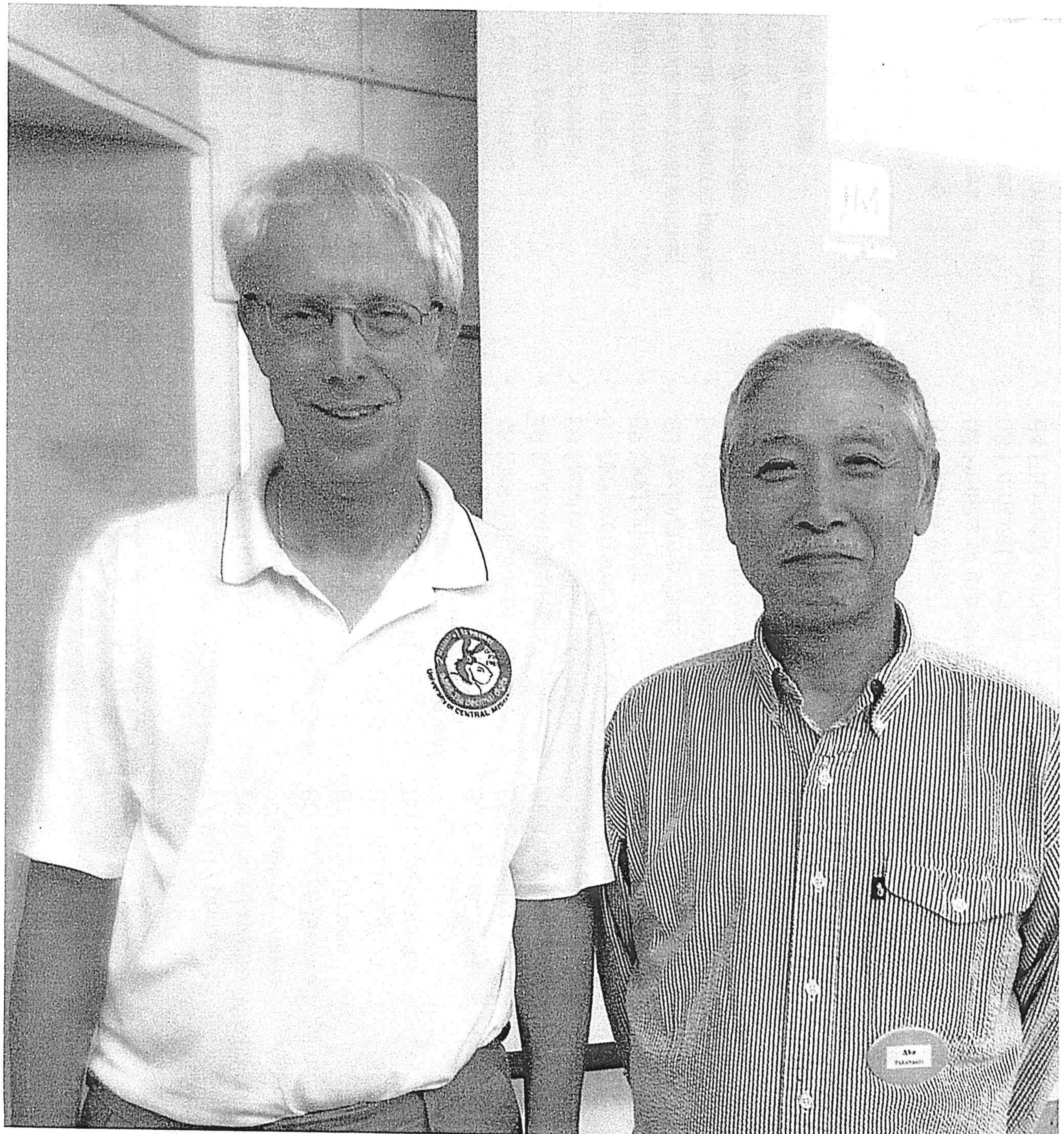
As of today, the largest known prime number is 2^74,207,281 – 1, a number with 22,338,618 digits.

The Electronic Frontier Foundation

is offering $150,000 to anyone who discovers a 100 million digits prime number.

Prof. Curtis Cooper (Central MO State University)

1 billion digits Mersenne prime number is worth $250,000



Akehiko Takahashi
January 21, 2016 near O'Fallon ·
Allowed on Timeline

Curtis Cooper finds the largest prime number so fa
Nice work.

Tag Photo **Add Location** **Edit**

Like **Comment** · **Share**

Chuck Steimel, Anthony D'Arienzo and 60 others

2 shares 4 Comme

Mark McCutchan Keep looking, I bet you can find a bigger one, Mr. T.
Like · Reply · January 21, 2016 at 8:39am

Tod Schattgen The suspense is killing me. What's the number?
Like · Reply · January 21, 2016 at 8:59am

Mark McCutchan Good story:
http://www.engadget.com/.../the-worlds-largest-prime.../

The world's largest prime number has...

ENGADGET.COM

Like · Reply · Remove Preview · January 21, 2016 at 12:30pm

Akehiko Takahashi He and his friends hope to discover a 100 million-digit prime number, which could net them $150,000 from the Electronic Frontier Foundation. Good will hunting.
Like · Reply · 1 · January 22, 2016 at 6:28pm

List of Notable Primes

- 1 Additive primes
- 2 Annihilating primes
- 3 Bell number primes
- 4 Carol primes
- 5 Centered decagonal primes
- 6 Centered heptagonal primes
- 7 Centered square primes
- 8 Centered triangular primes
- 9 Chen primes
- 10 Circular primes
- 11 Cousin primes
- 12 Cuban primes
- 13 Cullen primes
- 14 Dihedral primes
- 15 Double factorial primes
- 16 Double Mersenne primes
- 17 Eisenstein primes
- 18 Emirps
- 19 Euclid primes
- 20 Even prime
- 21 Factorial primes
- 22 Fermat primes
- 23 Fibonacci primes
- 24 Fortunate primes
- 25 Gaussian primes
- 26 Generalized Fermat primes
- 27 Genocchi number primes
- 28 Gilda's primes
- 29 Good primes
- 30 Happy primes
- 31 Harmonic primes
- 32 Higgs primes for squares
- 33 Highly cototient number primes
- 34 Irregular primes
- 35 (p, p–5) irregular primes
- 36 (p, p–9) irregular primes
- 37 Isolated primes
- 38 Kynea primes
- 39 Left-truncatable primes
- 40 Leyland primes
- 41 Long primes
- 42 Lucas primes
- 43 Lucky primes
- 44 Markov primes
- 45 Mersenne primes
- 46 Mersenne prime exponents
- 47 Mills primes
- 48 Minimal primes
- 49 Motzkin primes
- 50 Newman–Shanks–Williams primes
- 51 Non-generous primes
- 52 Odd primes
- 53 Padovan primes
- 54 Palindromic primes
- 55 Palindromic wing primes
- 56 Partition primes
- 57 Pell primes
- 58 Permutable primes
- 59 Perrin primes
- 60 Pierpont primes
- 61 Pillai primes
- 62 Primes of the form $n4 + 1$
- 63 Primeval primes
- 64 Primorial primes
- 65 Proth primes
- 66 Pythagorean primes
- 67 Prime quadruplets
- 68 Primes of binary quadratic form
- 69 Quartan primes
- 70 Ramanujan primes
- 71 Regular primes
- 72 Repunit primes
- 73 Primes in residue classes
- 74 Right-truncatable primes
- 75 Safe primes
- 76 Self primes in base 10
- 77 Sexy primes
- 78 Smarandache–Wellin prime
- 79 Solinas primes
- 80 Sophie Germain primes
- 81 Star primes
- 82 Stern primes
- 83 Super-primes
- 84 Supersingular primes
- 85 Swinging primes
- 86 Thabit number primes
- 87 Prime triplets
- 88 Twin primes
- 89 Two-sided primes
- 90 Ulam number primes
- 91 Unique primes
- 92 Wagstaff primes
- 93 Wall-Sun-Sun primes
- 94 Wedderburn-number primes
- 95 Weakly prime numbers
- 96 Wieferich primes
- 97 Wilson primes
- 98 Wolstenholme primes
- 99 Woodall primes

Prime Numbers

*Prime numbers are numbers whose divisors are given by exactly 1 and the number itself.

*There is no known useful formula that yields all of the prime numbers.

*Leonhard Euler discovered that the equation $f(n) = n^2 + n + 41$ produced primes consecutively when fed with numbers n= 0 to 39. This equations works only up to n=39.

*The fundamental theorem of arithmetic establishes the central role of primes in number theory

*Euclid demonstrates that there are infinite number of primes.

*Prime numbers are used in public-key cryptography

*Because of the importance in encryption algorithms such as RSA encryption, prime numbers can be important commercial commodities. In fact, R. Schlafly (1994) has obtained U.S. Patent 5373560 on the two primes (expressed in hexadecimal notation)

*twin prime conjecture says that there are infinitely many pairs of primes whose difference is 2.

*Goldbach's conjecture asserts that every even integer n greater than 2 can be written as a sum of two primes

*It is conjectured that there are infinitely many Fibonacci primes and Mersenne primes, but not Fermat primes

*In 2009, the Great Internet Mersenne Prime Search project was awarded a US$100,000 prize for first discovering a prime with at least 10 million digits. The Electronic Frontier Foundation also offers $150,000 and $250,000 for primes with at least 100 million digits and 1 billion digits, respectively.

*The oldest example to find prime numbers is the sieve of Eratosthenes which is useful for relatively small primes. The modern sieve of Atkin is more complicated, but faster when properly optimized.

* Mersenne numbers are numbers of the form $M_n = 2^n - 1$, giving the first few as 1, 3, 7, 15, 31, 63, 127, ... Interestingly, the nth Mersenne number is simply a string of n 1s when represented in binary. For example, $M_n = 2^n - 1 = 127 = 1111111_2$ is a Mersenne number. In fact, since 127 is also prime, 127 is also a Mersenne prime.

So many primes, but
So little time!

Let's look at some primes just for fun.

Emirps... 13, 17, 31, 37, 71, 73, 79, 97, 107, 113, 149, 157...

Sexy Primes... (5,11), (7,13), (11,17), (13,19), (17,23), (23,29), (31,37),...

Swinging Primes... Primes which are within 1 of a swinging factorial: $n≀ \pm 1$. 2, 3, 5, 7, 19, 29, 31, 71, 139, 251, 631, 3433,,,,

Annihilating Primes... Whatever they are, let's stay away from them. Because they will destroy you.

Happy Primes... a number that is both happy and prime. 7, 13, 19, 23, 31, 79, 97, 103, 109, 139, 167, 193,

379009... What is so unique about this prime?

Strobogrammatic prime... 11,101, 181, 619,16091, 18181

867-5309... Jenny's phone number in Tommy Tutone's hit song. 5309 as well as 8675309 are prime.

613... You can get different classes of numbers by rearranging its digits. 136 (triangular number) and the square of a number $361=19^2$

The Sieve of Eratosthenes

FEBRUARY 2, 2015 ISSUE The New Yorker
THE PURSUIT OF BEAUTY By Alec Wilkinson
Yitang Zhang solves a pure-math mystery on prime numbers.
http://www.newyorker.com/magazine/2015/02/02/pursuit-beauty

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Other methods: The Sieve of Atkins, The Sieve of Sundaram

1) Goldbach Conjecture

One of the oldest & best known unsolved problems in mathematics

Every **even integer** greater than 2 can be expressed as the sum of two primes.

$$6 = 3 + 3$$
$$8 = 5 + 3$$
$$10 = 5 + 5 = 7 + 3$$

2) Vinogradov's Three Primes Theorem

Any sufficiently large **odd integer** can be written as the sum of three prime numbers.

$$7 = 2 + 2 + 3$$
$$9 = 3 + 3 + 3 = 2 + 2 + 5$$
$$11 = 3 + 3 + 5$$
$$13 = 3 + 3 + 7$$
$$15 = 3 + 5 + 7$$

Luhn Algorithm mod 10

Most credit or debit cards have 16 digits. The first digit on a Mastercard is 5, on a Visa card is 4. On American Express is 3. Most gasoline cards starts with 7.

The test for the validity of a bank card number was invented in 1954 by German IBM scientist Hans Peter Luhn. Luhn's algorithm is a simple bit of arithmetic designed not to guard against malicious attacks, <u>but to protect against accidental transposing errors.</u>

This is how it works.

<u>4</u> 4 <u>3</u> 7 <u>1</u> 2 <u>1</u> 4 <u>5</u> 6 <u>1</u> 8 <u>9</u> 1 <u>7</u> 3

Underline the first digit and every alternative numbers. Double all the underline numbers.

8 6 2 2 10 2 18 14

Add all the numbers that are not underlined and the doubled numbers.

4 +7 + 2 + 4 + 6 + 8 +1 + 3 + 8 + 6 + 2 + 2 + 1 + 0 + 2 +1 + 8 +1 + 4 = 70

since 70 is divisible by 10, the number is correct.

American Express cards have only 15 digits. Underscore the second digit and every alternative numbers. For example

3 <u>7</u> 8 <u>2</u> 8 <u>2</u> 2 <u>4</u> 6 <u>3</u> 1 <u>0</u> 0 <u>0</u> 5

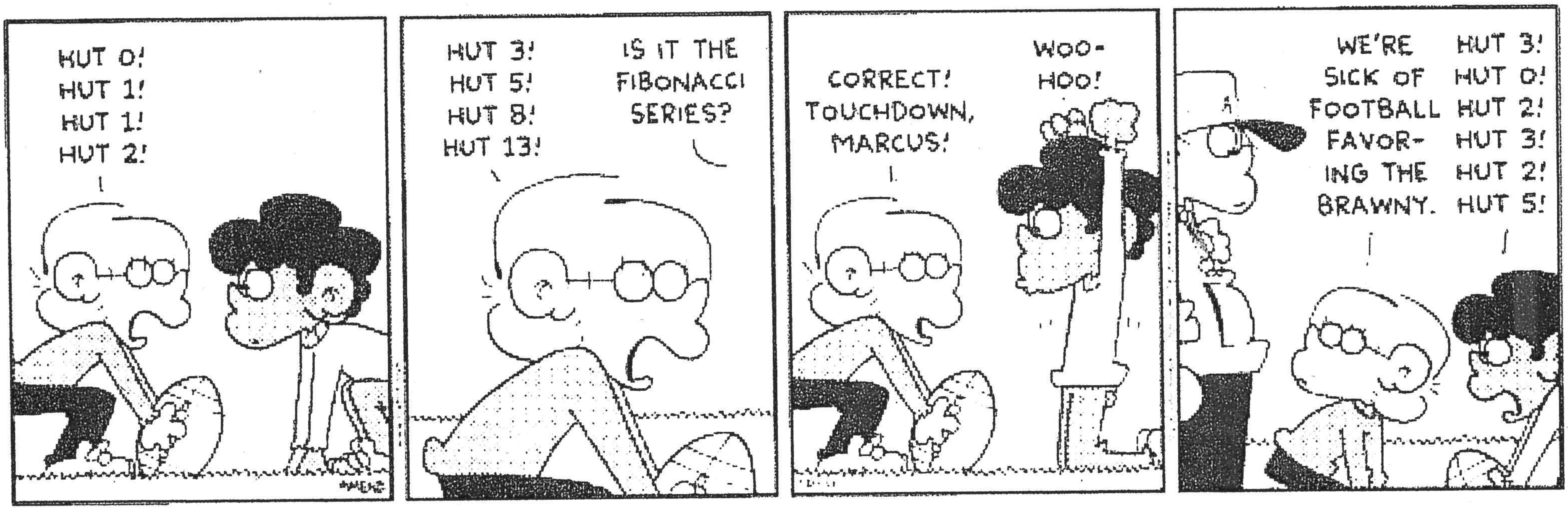
Double all the underlined numbers

(1 + 4) + 4 + 4 + 8 + 6+ 0 = 27 add to non-underlined numbers.

3 + 8 + 8 + 2 + 6 + 1 + 0 + 5 = 33 27 + 33 = 60 which is divisible by 10.

The Luhn algorithm will detect any single-digit error, as well as almost all transpositions of adjacent digits. It will not, however, detect transposition of the two-digit sequence *09* to *90* (or vice versa).

More complex check-digit algorithms (such as the <u>Verhoeff algorithm</u> and the <u>Damm algorithm</u>) can detect more transcription errors.



3, 0, 2, 3, 2, 5, 5, 7, 10, 12, 17, 22, ?

What is the next number for this sequence?

1 2 4 8 16 ?

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

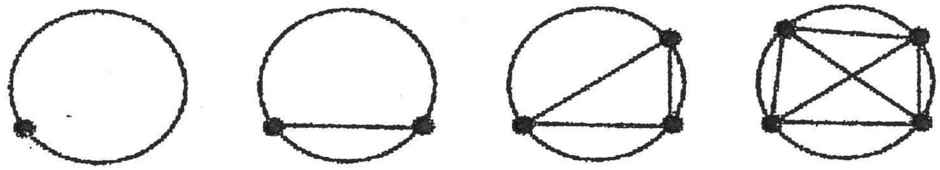
1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

# of points on a circle and # of partitioned regions

Moser's Circle Problem



Moser's circle problem asks to determine the number of pieces into which a circle is divided if n points on its circumference are joined by chords with no three internally concurrent.

$$g(n) = \binom{n}{4} + \binom{n}{2} + 1$$

1, 2, 4, 8, 16, ?, ...

geometric series says

1, 2, 4, 8, 16, 32, 64,128, 256, ...

Moser says

1, 2, 4, 8, 16, ?, 57, 99,...

some people can show

1, 2, 4, 8, 16, ?, ...

lines rather than chords

2, 4, 7, 11, 16, 22, 29,...

1, 2, 4, 8, 16, ___ 30 32

the number of divisors based on factorials sequence

1!	2!	3!	4!	5!	6!
1	2	4	8	16	30

3! = 3 x 2 x 1 = 6

numbers of divisors = 4

6,3,2,1

4! = 4 x 3 x 2 x 1 = 24

numbers of divisors = 8

24, 12, 8, 6, 4, 3, 2, 1

5! = 5x4x3x2x1 = 120

number of divisors = 16

6! = 6x5x4x3x2x1 = 720

number of divisors = 30

Factorial

$4! = 4 \times 3 \times 2 \times 1$ $0! = 1!$ by definition

change from L4 to 4! In 1808 by C. Kramp

double factorial $9!! = 9 \times 7 \times 5 \times 3 \times 1$

primorial $8\# = 7 \times 5 \times 3 \times 2$

superfactorial $sf(4) = 1! \times 2! \times 3! \times 4!$

Factorion = the sum of the factorials of its digits

e.g. , 145 is a factorion because $1! + 4! + 5! = 1 + 24 + 120 = 145$.

There are just four factorions (in base 10) and they are 1, 2, 145 and 40585 .
"Factorion" was coined by Clifford A. Pickover.

$4! = 24$ but $!4$ (subfactorial) $= 9$ (de Montmort number)

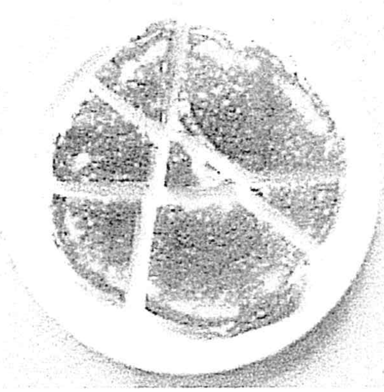
What is the next two numbers of the sequence?

1, 2, 4, 7, 11, 16, ____ , ____

challenge:

Can you put these numbers in a single column and form a triangle using other numbers?

This sequence is called the Lazy Caterer's sequence.



Lazy Caterer's Sequence

1											
2	3										
4	5	6				Triangular numbers					
7	8	9	10								
11	12	13	14	15							
16	17	18	19	20	21		Floyd Triangle				
22	23	24	25	26	27	28					
29	30	31	32	33	34	35	36				
37	38	39	40	41	42	43	44	45			
46	47	48	49	50	51	52	53	54	55		
56	57	58	59	60	61	62	63	64	65	66	
67	68	69	70	71	72	73	74	75	76	77	78

The Luhn Algorithm

It is a checksum formula (mod 10) used to validate a variety of identification numbers. It was created by a German IBM scientist Hans Peter Luhn. It is not intended to be a cryptographically secure hash function; it was designed to protect against accidental errors like mistyped incorrect numbers.

double all the underlined numbers

4 4 3 7 1 2 1 4 5 6 1 8 9 1 7 3

8 6 2 2 10 2 18 14

add all the numbers not underlined and doubled numbers

$4 + 7 + 2 + 4 + 6 + 8 + 1 + 3$
$+ 8 + 6 + 2 + 2 + 1 + 0 + 2$
$+ 1 + 8 + 1 + 4 = 70$

Most credit or debit cards have 16 digits. American Express has only 15 digits. Underline the 2nd digit and every other one and follow the same rule. The Luhn Algorithm detects only any single-digit error. The improvements were made by the Verhoeff algorithm and the Damm algorithm.

The first digits: Visa card = 4, MasterCard = 5 American Express = 3 Gas card = 7

Check Digit Algorithm

Check digit algorithms are generally designed to capture human transcription errors. In order of complexity, these include the following:

- single digit errors, such as 1 → 2
- transposition errors, such as 12 → 21
- twin errors, such as 11 → 22
- jump transpositions errors, such as 132 → 231
- jump twin errors, such as 131 → 232
- phonetic errors, such as 60 → 16 ("sixty" to "sixteen") or 50 →15

digital sum mod 10 for most credit cards

weighted sum modulo 10 (UPS, Bank Routing Transit Numbers)

The ISBN-10 code uses mod 11

International Bank Account Number uses letters plus two digit codes (Verhoeff algorithm and mod 97)

$\sqrt{625} = 25$ then

what is dr(625)?

Formula:

$dr(n) = n - 9\lfloor (n-1)/9 \rfloor$

$\lfloor x \rfloor$ = floor function

dr(625) =

$625 - 9\lfloor (625 - 1)/9 \rfloor =$

$625 - 9\lfloor (624)/9 \rfloor =$

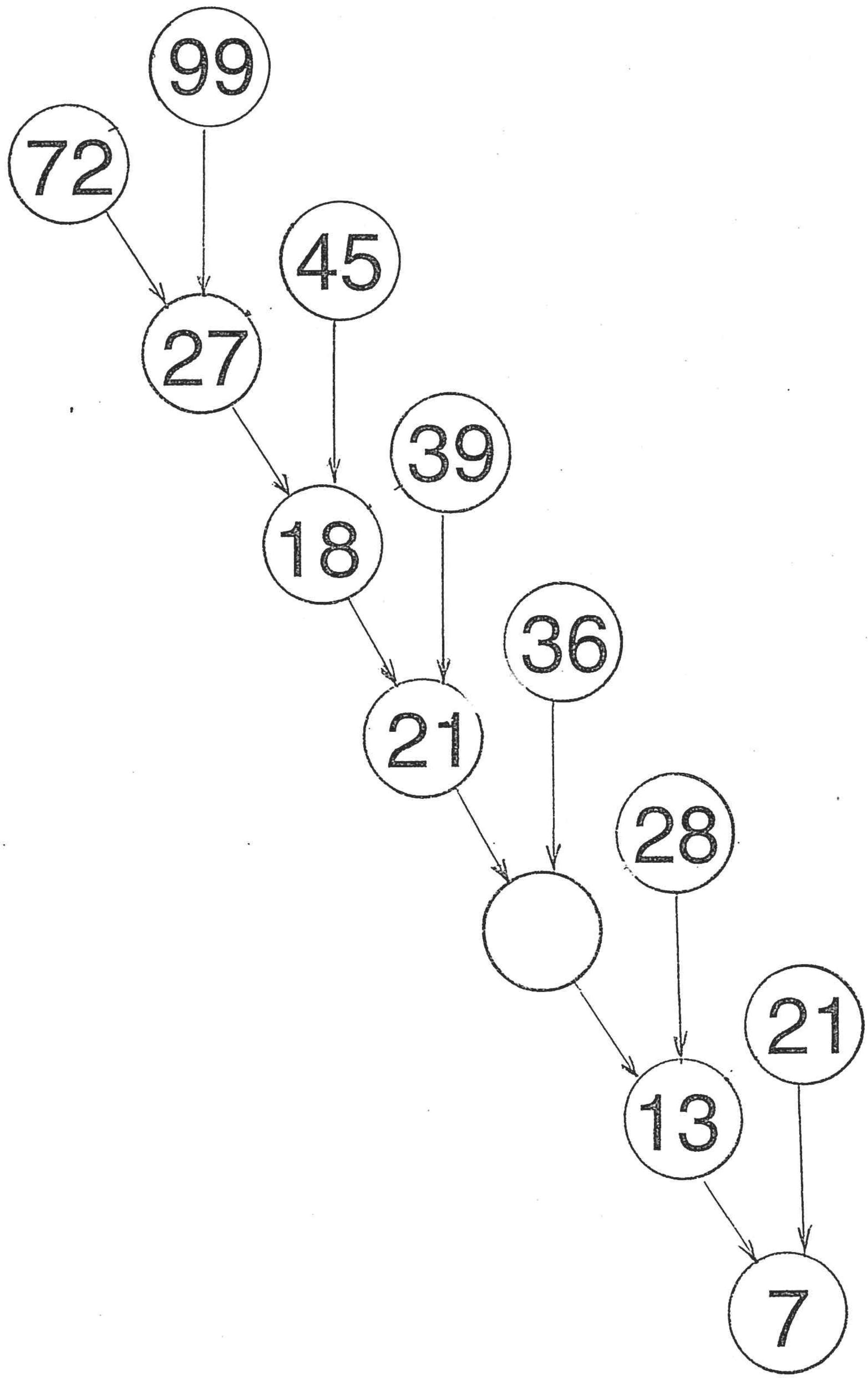
$625 - 9\lfloor (69.333... \rfloor =$

$625 - 9\lfloor (69) \rfloor$

$= 625 - 621 = 4$

so dr(625) = 4

Find dr(89) =


99
72
45
27
39
18
36
21
28
21
13
7

Grandi's Series

What is the sum of the following?

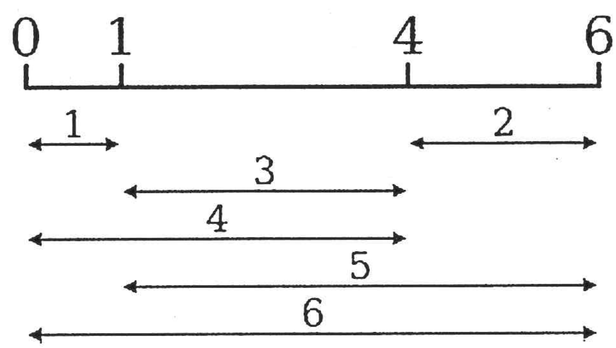
$$1 - 1 + 1 - 1 + 1 - 1 + 1 \ldots = ?$$

The answer could be

0, 1, or 1/2

1/2 is called Cesaro sum.
Euler had his own proof.

Golomb Ruler



a Golomb ruler is a set of marks at integer positions along an imaginary ruler such that no two pairs of marks are the same distance apart.

The Golomb ruler was named for Solomon Wolf Golomb. (1932-2016) He is an American mathematician and engineer and a professor of electrical engineering at the USC, best known to the general public and fans of mathematical games as the inventor of polyominoes, the inspiration for the computer game ***Tetris***. He was also the inventor of Golomb coding. It is a lossless data compression method. He received a B.A. from Johns Hopkins and an M.A. and a Ph.D. in mathematics from Harvard in 1957 with a dissertation on "Problems in the Distribution of the Prime Numbers". He received the medals from the US Security Agency and from the Russian Academy of Science and a Hamming Medal from IEEE. He was a great metagrobologist.

Information Theory/Error Correction...used for error correcting codes

Radio Frequency Selection...used in the selection of radio frequencies to reduce the effects of intereference

Radio Antennae Placement...used in the design of phased radio antennas such as radio telescopes, Golomb ruler configuration can be seen at cell sites

Current Transformers: Multi-ratio current transformers use Golomb rulers to place transformer tap points.

Find two order 5 Golomb Rulers (that is from 0 to 11).

Tetration

was introduced by Goldstein in 1947. It was derived from tetra and literation. It is referred to as "hyperpower or superexponentiation"

standard notation ^{n}a

Knuth notation $a \uparrow\uparrow n$

Conway notation $a \rightarrow n \rightarrow 2$

$2^4 = 16$, but $^{4}2 = 2^{16} = 65{,}536$

$5^3 = 5 \times 5 \times 5 = 125$ $\quad$ $^{3}5 = 5^{5^{5}} = \text{huge}$

$$^{n}a = a^{a^{a^{\cdot^{\cdot^{\cdot}}}}}$$

a is exponentiated by itself n times

Infinite tower of powers $x\uparrow\uparrow\infty$

$$2 = x^{x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}}} \qquad\qquad 4 = x^{x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}}} \quad \text{then } 4 = x^4$$

$2 = x^2 \quad x = \sqrt{2}$ $\qquad$ $(4)^{1/4} = (x^4)^{1/4}$ then $x = \sqrt{2}$

Does this mean $x\uparrow\uparrow\infty = 2$

$x\uparrow\uparrow\infty = 4$ they are both correct?

The most famous number comes into play in this problem. Can you name this number?

3 Conway Sequence 2

1, 3
1, 1, 1, 3
3, 1, 1, 3
1, 3, 2, 1, 1, 3
1, 1, 1, 3, 1, 2, 2, 1, 1, 3
?

1, 2
1, 1, 1, 2
3, 1, 1, 2
1, 3, 2, 1, 1, 2
1, 1, 1, 3, 1, 2, 2, 1, 1, 2
?

This sequence gave rise to John Conway's creation of **the Game of Life**. It is a cellular automaton and its evolution is determined by its initial state requiring no further input. Within the grid of square cells, they are either alive or dead. Every cell interacts with its eight neighbors, which are the cells that are horizontally, vertically, or diagonally adjacent based on the following rules:

1. Any live cell with **fewer than two** live neighbors dies, as if caused by underpopulation.
2. Any live cell with **two or three** live neighbors lives on to the next generation.
3. An live cell with **more than three** live neighbors dies, as if by overpopulation.
4. Any dead cell with **exactly three** live neighbors becomes a live cell, as if by reproduction.

The game made Conway instantly famous, but it also opened up a whole new field of mathematical research, the field of cellular automata ... Because of Life's analogies with the rise, fall and alterations of a society of living organisms, it belongs to a growing class of what are called "simulation games"

Ever since its publication, Conway's Game of Life has attracted much interest, because of the surprising ways in which the patterns can evolve. Life provides an example of emergence and self-organization. Scholars in various fields, such as computer science, physics, biology, biochemistry, economics, mathematics, philosophy, and generative sciences have made use of the way that complex patterns can emerge from the implementation of the game's simple rules.

Collatz Conjecture (hotpo)

also called the 3n + 1 conjecture, the Ulam conjecture, Kakutani's problem, the Thwaites conjecture , Hasse's algorithm , the Syracuse problem. <u>Paul Erdős</u> said about this conjecture: "Mathematics may not be ready for such problems." He also offered $500 for its solution.

6, 3, 5, 8, 4, 2, 1

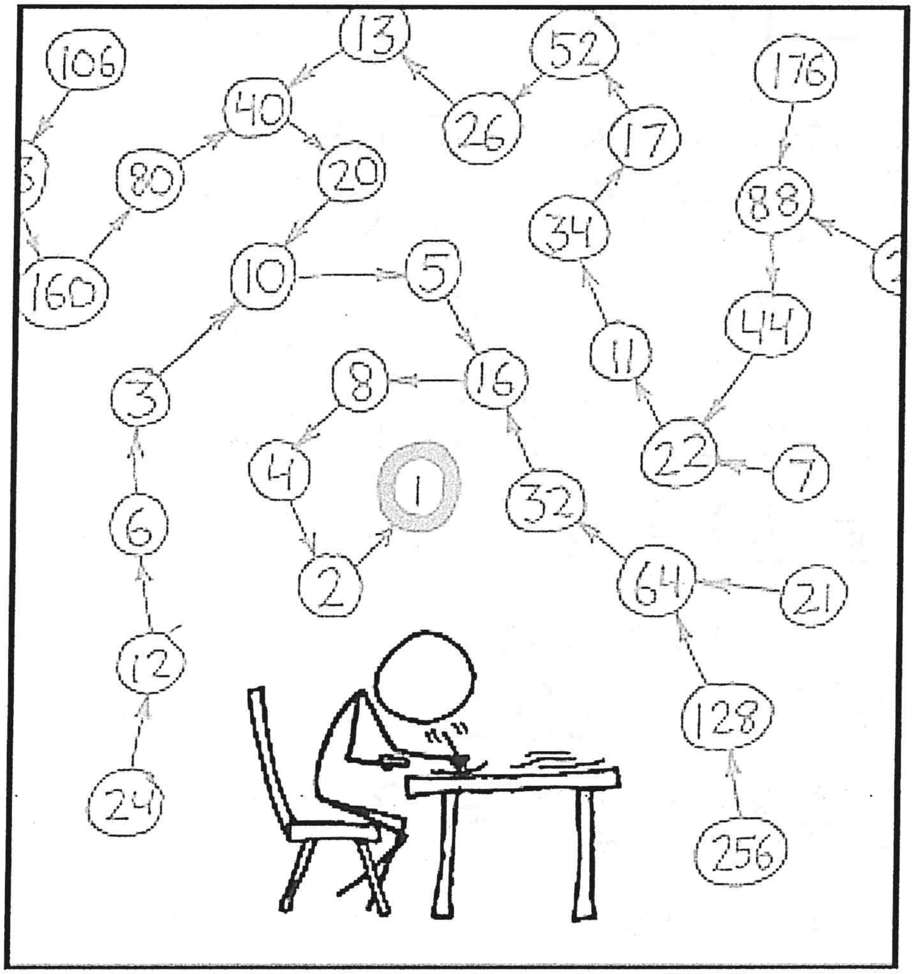
7, 11, 17, 28, 13, 20, 10, 5, 8, 4, 2, 1

10, 5, 16, 8, 4, 2, 1

11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1

12, 6, 3, 10, 5, 8, 4. 2, 1

15, ?______________________________



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Vedic Math

squaring the two digit numbers that end in 5

$35^2 = 1225 \qquad 75^2 = ?$

Squaring the two digit number starting with 5

$52^2 = 2704 \qquad 56^2 = ?$

Multiplying two digit numbers where the first figures are the same and the last figures add up to 10

32 x 38 = 1216

43 x 47 = ?

Multiplying a number by 11

26 x11 = 281

12543 x 11 = ?

Multiplying a number by 111

123 x111 = 13653

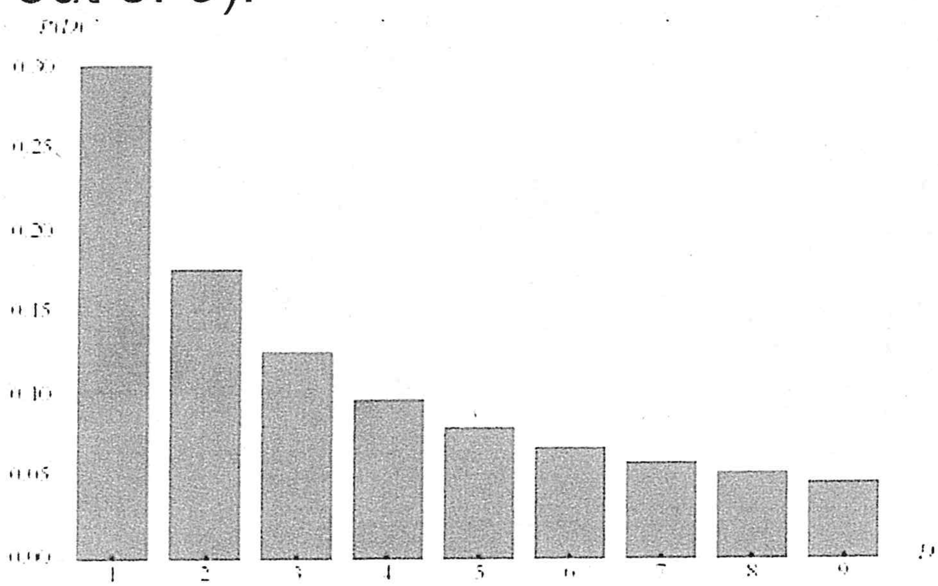
143 x 111 = ?

Multiplying two numbers near 100 but both below 100

88 x 98 = 8624

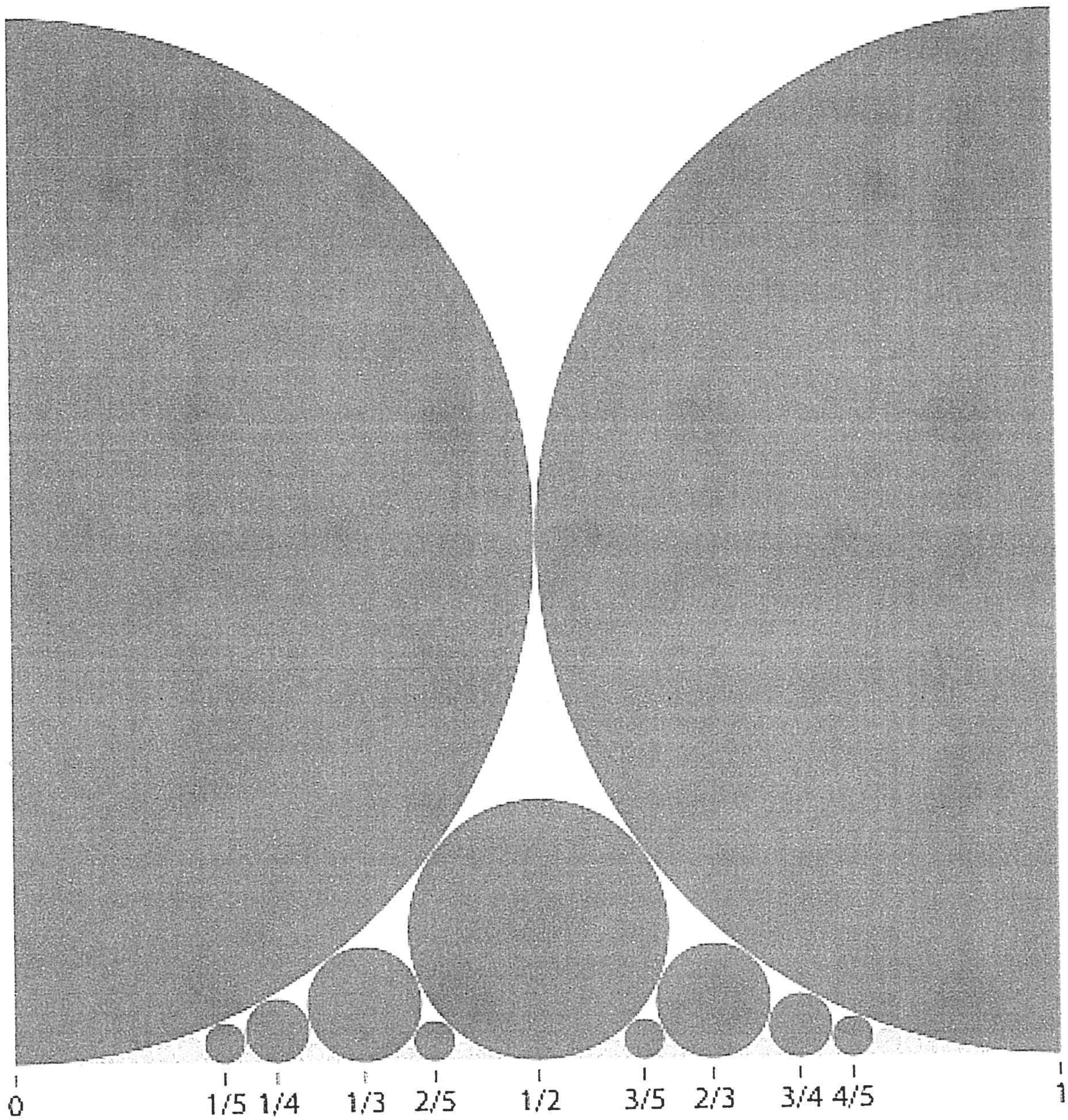
	88	12
x	98	2
	86	24

Benford's law, also called the first-digit **law**, is an observation about the frequency distribution of leading digits in many real-life sets of numerical data. This **law** states that in many naturally occurring collections of numbers, the leading significant digit is likely to be small. Benford's law shows that in listings, tables of statistics, randomly selected street addresses, etc., the digit 1 tends to occur with probability ~30%, much greater than the expected 11.1% (i.e., one digit out of 9).



P= log (1 + 1/d)

Farey Sequence, Ford Circles, Mediant Fractions



Sabermetrics

As originally defined by Bill James in 1980, sabermetrics is "the search for objective knowledge about baseball", based on statistical analysis in game activity. James coined the phrase in part to honor the Society for American Baseball Research.

Sicherman's Difference Triangle

examples with first six digits

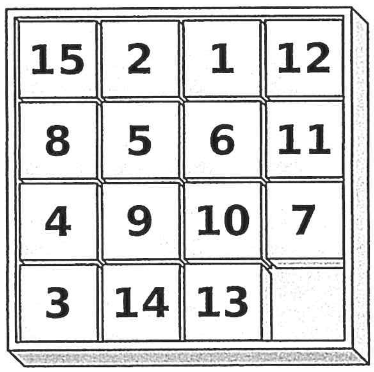
6 1 4	6 5 2	4 6 1	6 2 5	5 6 2
5 3	1 3	2 5	4 3	1 4
2	2	3	1	3

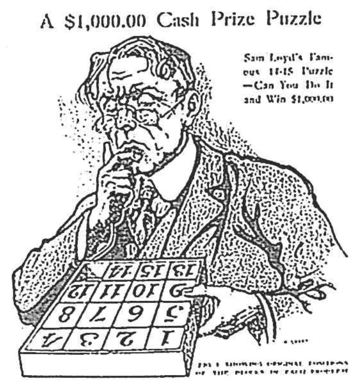
Form difference triangles using
1, 2, 3, 4, 5, 6, 7, 8, 9,10

Mirror images are not acceptable. A chiral object and its mirror image are said to be enantiomorphs. Human hands are perhaps the most universally recognized example of chirality.

a pharmacological importance: same elements but different molecular structures

15 puzzle

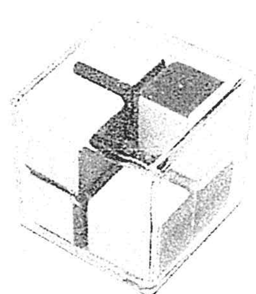




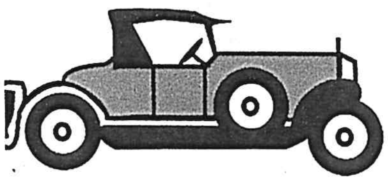
It is a sliding puzzle that consists of a frame of numbered square tiles in random order with one tile missing. To solve the puzzle, the numbers must be rearranged into order. The puzzle was invented by Noyes Palmer Chapman, a postmaster in Canastota, New York. He had applied for a patent on February 21, 1880, but the patent was later rejected. The game became a craze all over the world.

Sam Loyd claimed from 1891 until his death in 1911 that he invented the puzzle. Some later interest was fueled by Loyd offering a $1,000 prize for anyone who could provide a solution. for achieving a particular combination specified by Loyd, namely reversing the 14 and 15.

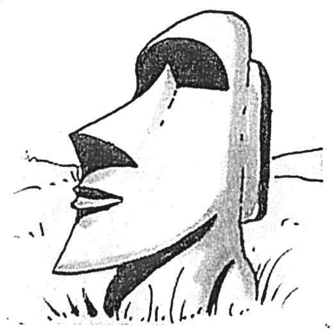
The "Minus Cube" which was the 3D form of sliding puzzle was invented in Russia. This cube along with Piet Hein's SOMA cube gave rise to the creation of Rubik's Cube. Piet Hein's pen name was tombstone. He said: <u>A problem worthy of attack proves its worth by fighting back.</u>



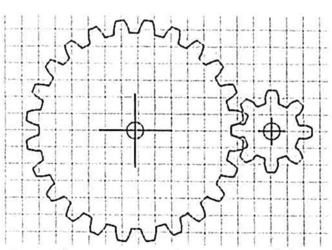
1) You have 100 lbs of potatoes, which are 99 percent water by weight. You let them dehydrate until they're 98 percent water. How much do they weigh now?



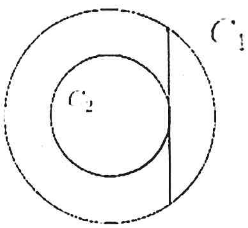
2) You drive from point A to point B at the rate of 60 mph and return the same route at the rate of 40 mph What is the rate of the average mph for the entire trip?



3) Worshipful natives are rolling a giant statue across their island. They made an idol statue of you because they think you are a Math God. The statue rests on a slab, which rests on rollers that have a circumference of 1 meter each. How far forward will the statute have moved when the rollers have made 1 revolution? If you fail to get this EASY answer, the math-worshipping cannibals will be very upset and eat you alive.



4) How many times must an 8-toothed cogwheel rotate on its axis to circle around a 24-toothed cogwheel?



5) There are two concentric circles. The radius of the larger circle is R and the radius of the smaller circle is r. The length of the chord tangent to the inner circle is 20 inches. Find the area of the annulus.

Mathematics is Music for the Mind and Music is Mathematics for your Soul. Years may wrinkle our skin, but the love of math will give us the enduring power to live and enjoy life and the loss of enthusiasm for math wrinkles our soul. Ancient Greeks called eudaimonia, or a life composed of all the highest goods. Professor Su, the former President of MAA, said of five basic human desires that are met through the pursuit of mathematics:

play, beauty, truth, justice, and love.

The beauty of math helps us want to become the dreamer of the dreams. In seeking truth and justice we we are compelled to be movers and shakers of the world. Through play we can enhance and enjoy our life. With love we hope to be the music makers of mathematics. So let's go out and aspire to inspire others before we expire. Together we can make this world a better place through mathematics.