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\documentclass[a4paper]{article}

\usepackage[english]{babel}
\usepackage[utf8]{inputenc}
\usepackage{amsmath}
\usepackage{graphicx}
\usepackage[colorinlistoftodos]{todonotes}

\title{Probability Paradox\\
Washington University Math Circle}
\author{Chang Liu}

\date{September 24, 2017}

\begin{document}
\maketitle


\section{Birthday Problem}

\begin{itemize}
\item Suppose you flip a coin and bet that it will come up tails. Since you are equally likely to get heads or tails, the probability of tails is $50\%$. This means that if you try this bet often, you should win about half the time.
\item \textbf{What if somebody offered to bet that at least two people in your math class had the same birthday? Would you take the bet?}
\item This question is more complicated than flipping a coin, because the chance of finding two people with the same birthday depends on the number of people you ask. If there were only one other person in your math class, you might be surprised to find out that she had the same birthday as you. If there were a pair of people with the same birthday in a class of 366 people, would you still be surprised?
\item \textbf{How large must a class be to make the probability of finding two people with the same birthday at least $50\
%$?}
\item Let's forget about leap year when we solve this problem (no February 29 birthdays!) This way, we can assume that a year is always 365 days long.
\item Also, let's assume that a person has an equal chance of being born on any day of the year, even though some birthdays may be slightly more likely than others. That will simplify the math, without changing the result significantly.
\item \textit{\textbf{(challenging)}} What's the probability that someone in the room have the same birthday as you?
\end{itemize}


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\section{Monty Hall Problem}
\begin{itemize}
\item There are 3 doors, behind which are two goats and a car.
\item You pick a door (call it door A). You?re hoping for the car of course.
\item Monty Hall, the game show host, examines the other doors (B and C) and always opens one of them with a goat (Both doors might have goats; he?ll randomly pick one to open).
\item \textbf{Here?s the game: Do you stick with door A (original guess) or switch to the other unopened door? Does it matter?}
\item What about the following cases?
 \begin{enumerate}
   \item The host offers the option to switch only when the player's initial choice is the winning door.
   \item The host offers the option to switch only when the player has chosen incorrectly.
   \item The host knows what lies behind the doors, and (before the player's choice) chooses at random which goat to reveal. He offers the option to switch only when the player's choice happens to differ from his.
   \item \textit{\textbf{(Challenging)} }What if there are N doors, and the host opens p losing doors and then offers the player the opportunity to switch?
  \end{enumerate}
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\end{itemize}



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\section{Gambler's Ruin}

\begin{itemize}
\item A gambler has a certain amount of money ("B") and is playing a game of chance with some win probability less than 1. Every time he wins, he raises his stake to a certain fraction, 1/N, of his bankroll, where N is a positive number. The gambler doesn't reduce his stake when he loses.
\item Every time he wins, he'll raise his stake to  \$B/N, or his bankroll divided by N. When B= \$1000 and N=4, for example, he'll gamble \$250 each time going forward. Should he win, he'll raise it again. Should he lose, he'll keep his stake at \$250.
\item \textbf{If he keeps at it, what are his expected winnings?}
\item \textit{(Don't need to answer that, still an open question)} What about N players with initial capital $x_1, x_2, \cdots, x_n$ dollars, respectively, play a sequence of (arbitrary) independent games and win and lose certain amounts of dollars from/to each other according to fixed rules. The sequence of games ends as soon as at least one player is ruined.
\end{itemize}


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\section{Abraham Wald's Memo}
\begin{itemize}
\item Abraham is tasked with reviewing damaged planes coming back from sorties over Germany in the Second World War. He has to review the damage of the planes to see which areas must be protected even more.
\item Abraham finds that the fuselage and fuel system of returned planes are much more likely to be damaged by bullets or flak than the engines.
\item \textbf{What should he recommend to his superiors?}
\end{itemize}




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\section{Simpson's Paradox}

\begin{itemize}
\item The following is a study of gender bias among graduate school admissions to University of California, Berkeley. The admission figures for the fall of 1973 showed that men applying were more likely than women to be admitted, and the difference was so large that it was unlikely to be due to chance.
\begin{figure}[h]
  \centering
    \includegraphics[width=0.4\textwidth]{1.png}
\end{figure}
\item But when examining the individual departments, it appeared that six out of 85 departments were significantly biased against men, whereas only four were significantly biased against women. In fact, the pooled and corrected data showed a "small but statistically significant bias in favor of women." The data from the six largest departments is listed below.
\begin{figure}[h]
  \centering
    \includegraphics[width=0.5\textwidth]{2.png}
\end{figure}
\item \textbf{Do you think there are gender biases among graduate school admissions?}

\end{itemize}
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\section{Bonus}
If you finish early, you can try here.
\begin{enumerate}
\item A gambler goes to bet. The dealer has 3 dice, which are fair, meaning that the chance that each face shows up is exactly 1/6. The dealer says: "You can choose your bet on a number, any number from 1 to 6. Then I'll roll the 3 dice. If none show the number you bet, you'll lose \$1. If one shows the number you bet, you'll win \$1. If two or three dice show the number you bet, you'll win \$3 or \$5, respectively." \textbf{Is it a fair game?}
\item You are a prisoner sentenced to death. The Emperor offers you a chance to live by playing a simple game. He gives you 50 black marbles, 50 white marbles and 2 empty bowls. He then says, "Divide these 100 marbles into these 2 bowls. You can divide them any way you like as long as you use all the marbles. Then I will blindfold you and mix the bowls around. You then can choose one bowl and remove ONE marble. If the marble is WHITE you will live, but if the marble is BLACK... you will die." \textbf{How do you divide the marbles up so that you have the greatest probability of choosing a WHITE marble?}
\item "I'm a very rich man, so I've decided to give you some of my fortune. Do you see this bag? I have 5001 pearls inside it. 2501 of them are white, and 2500 of them are black. No, I am not racist. I'll let you take out any number of pearls from the bag without looking. If you take out the same number of black and white pearls, I will reward you with a number of gold bars equivalent to the number of pearls you took." \textbf{How many pearls should you take out to give yourself a good number of gold bars while still retaining a good chance of actually getting them?}
\item Mike and James are arguing over who gets the last cookie in the jar, so their dad decides to create a game to settle their dispute. First, Mike flips a coin twice, and each time James calls heads or tails in the air. If James gets both calls right, he gets the last cookie. If not, Mike picks a number between one and six and then rolls a die. If he gets the number right, he gets the last cookie. If not, James picks two numbers between one and five, then spins a spinner with numbers one through five on it. If the spinner lands on one of James' two numbers, he gets the last cookie. If not, Mike does.
\textbf{Who is more likely to win the last cookie, Mike or James? And what is the probability that person wins it? }
\end{enumerate}

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