Geometric Probability

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1   Warmup problems

1. A six-sided die is rolled, find the probability that an even number is obtained.

2. If a box contains two yellow balls and one red, what is the probability of drawing a red and a yellow if two balls are drawn?

3. Three six-sided dice are rolled. What is the probability that at least one comes up with a 5 or 6?

4. There is a 20% chance it will rain today. If it rains, there is a 10% chance that we will be allowed to go outside; otherwise, there is an 80% chance we will be able to go outside. What is the probability that we will be allowed to go outside?

2 Geometric probability

Geometric probability involves the distributions of length, area, and volume for geometric objects under stated conditions. The same basic concept behind probability applies, but instead of calculating total outcomes and particular outcomes, it often involves calculating total area and particular area of a geometric figure, and the resulted probability is calculated using the formula

$$P = \frac{\text{Area of the particular region}}{\text{Area of the total region}}.$$

Let's start with an example.

1. A point is chosen at random inside a circle if radius 2. What is the probability that the point is within one unit of the center of the circle?

2. A point is chosen at random inside a square which has side length 4. What is the probability that the point is within 1 inch of a side of the square?

3. In each of the following figures, find the probability that a randomly chosen point lies in the shaded region.
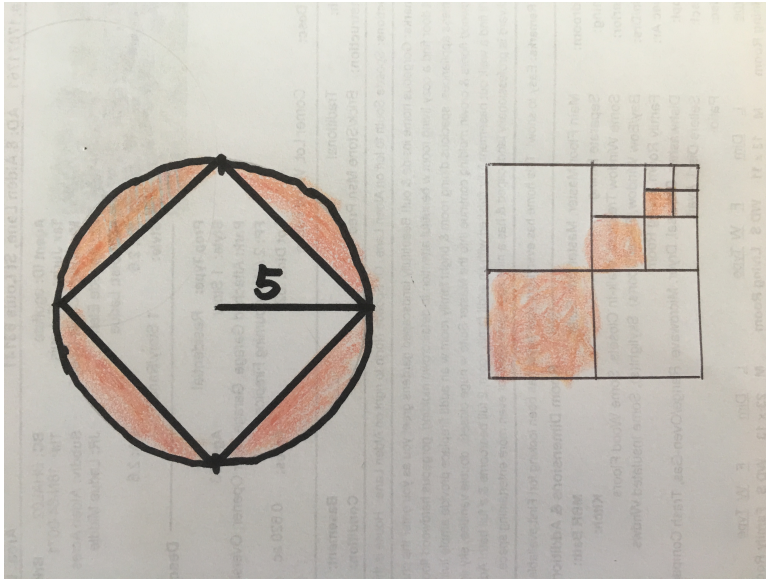
3 Challenging problems

Some probability problems do not involve geometry directly, but can be solved nicely using geometric representations. Let's start from playing a game.

4. You and I both randomly choose an integer from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. If your number is no more than 2 away from my number, you win $1,000! What is your probability of winning? (Assume you make wise choices.)

5. Let's change the game a little. Now instead of choosing the integers, you can I both randomly choose a real number in [1,10]. And the rule is still that, if your number is no more than 2 away from my number, you win. Now what is your probability of winning? (Assume you still make wise choices.)

6. A women arrives at an airport between 11 and 12 o'clock. she waits for her husband for 10 minutes and if he does not arrive, she hails a taxi (and he is in a lot of trouble!) Her husband shows up at the airport at some time between 11 and 11:50. He waits for 20 minutes and it his wife has not shown up, he goes home (and he is in a lot of trouble!). What is the probability that the woman rides home with her husband?

7. Two numbers (not necessarily integers) are chosen at random between 0 and 10. What is the probability that they differ by no more than 5?

8. A stick is broken at two random places. What is the probability that the three resulting pieces can form a triangle?