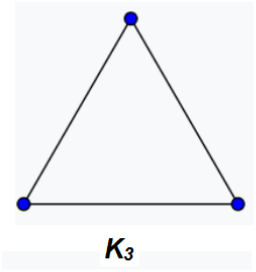
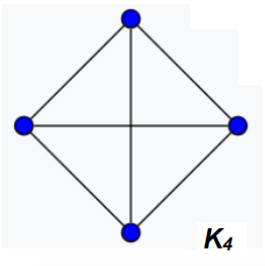
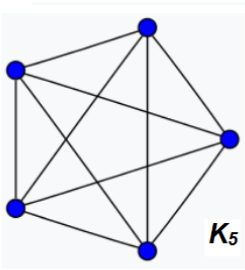
WUSTL Math Circle

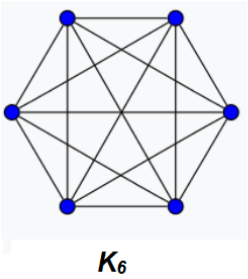
Order Appearing in Coloring Large Set of Data

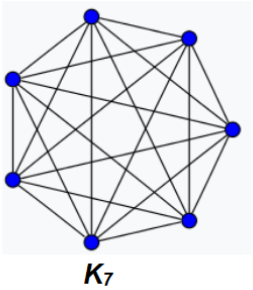
Here are some *complete graphs*.

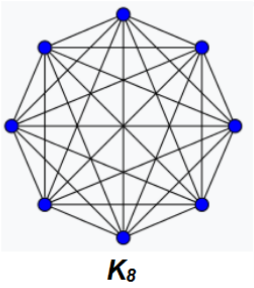








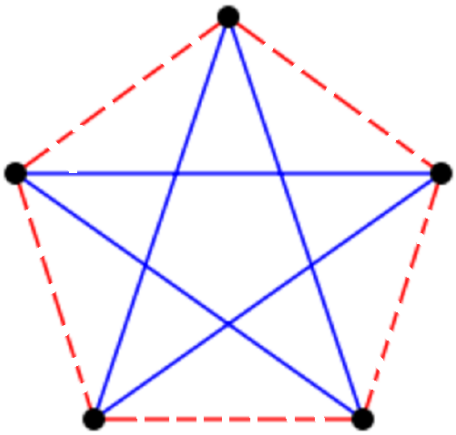




1. How many K_3 do you see inside K_4?

2. How many K_4 do you see inside K_6?

3. Here I have colored the edges of K_5 with two colors red and blue (in black-white printing, red edges are dashed and blue edges are solid) such that no monochromatic triangle appears.



Can you do the same with K_6?

4. Color the edges of K_6 with two colors say red and blue such that no red K_3 and no blue K_4 appear.

5. Color the edges of K_7 with two colors say red and blue such that no red K_3 and no blue K_4 appear.

6. Color the edges of K_8 with two colors say red and blue such that no red K_3 and no blue K_4 appear.

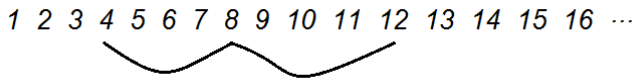
7. Can you color the edges of K_9 with two colors say red and blue such that no red K_3 and no blue K_4 appear?

8. Can you show that this is impossible?

9. Find the largest complete graph you can with a coloring of its edges with three colors say red, blue and yellow such that no monochromatic triangle appear.[1]

[1] World record is K_{16}.

We would say that $4, 8, 12$ form a 3-term arithmetic progression, because the distance between 4 and 8 equals the distance between 8 and 12.



10. Which of the following sequences are in arithmetic progression?

$$5, 14, 23$$

$$5, 14, 23, 33$$

$$5, 14, 23, 32$$

11. Here is a coloring of integers from 1 to 8 with two colors red and blue such that no monochromatic 3-term arithmetic progression appears.

1	2	3	4	5	6	7	8
R	B	B	R	R	B	B	R

Either do the same for integers from 1 to 9, or prove that this is impossible.

12. Color integers from 1 to 15 with three colors such that no monochromatic 3-term arithmetic progression appears.

13. Color integers from 1 to 20 with three colors such that no monochromatic 3-term arithmetic progression appears.[2]

[2]World record is 26.

14. Try to find your largest integer N, with a cloring of integers from 1 to N with two colors such that no monochromatic 4-term arithmetic progression appears.[3]

[3]World record is 34.

This mathematics is called *Ramsey Theory*, with slogan

TOTAL DISORDER IS IMPOSSIBLE.

More grown-up, or even now!, you could follow it from the book:

Graham, R. L., B. Rothschild, J. H. Spencer, *Ramsey Theory*, Second Edition, John Wiley and Sons, 1990.