The Game of Set

Feb 11th, 2018

1. Let S denote a standard SET deck. S contains one of each possible type of card. How many cards are there in S?

2. How many sets are there in S?

3. Prove the Fundamental Theorem of SET:

   Theorem 1 *Given two cards, $x, y \in S$; there is a unique card $z \in S$ for which (x, y, z) is a set.*

4. Given two set cards, x and y, define their **product**, $x \star y$, to be the unique card for which $(x, y, x \star y)$ is a set.

   What properties does this multiplication have? Is it commutative? Is it associative? Is there an identity?

5. There should be a correspondence between S and the integers $\{1, 2, ..., 81\}$ by writing the numbers in base 3. How does this correspondence work?

   Hint: Another way to think of this is each card can be though of as a vector (x_1, x_2, x_3, x_4) where each of the x_i are either 0,1 or 2. (How is this the same as thinking in base 3?)

6. By writing elements in S as vectors (or as numbers between 0 and 80 in base 3), determine the set product for these numbers.

   Hint: look at many examples and look at what happens in each component of the vector.

   (The set of vectors above is called Z_3^4).

7. Compute the following products:

   1. $(1, 0, 0, 0) \star (1, 0, 0, 1) =$
   2. $(0, 1, 0, 0) \star (0, 2, 0, 1) =$
   3. $(2, 2, 2, 2) \star (0, 2, 2, 1) =$
   4. $(0, 0, 0, 0) \star (1, 1, 1, 1) =$

8. Given a card $x \in S$, how many sets does x belong to?

9. Does your answer in Question 8 agree with your answer in Question 2?

10. What is the smallest stack such that every card must have a set in the stack? In other words, you want to deal o a number of cards, n, so that you know, without looking at the cards, that every one of those cards is in at least one set. How big must n be?

11. Now, what is the largest stack with no sets?

    Hint. First consider the game of Set with only 2 properties (9 cards in the deck), say shape and number.

12. (Open Question) You can generalize the game of SET by adding more Features, say k; or adding more options in each feature, say we used n shapes, colors, shadings, etc. Answer the rest of the questions with this new deck of Set cards. (Many of the questions, such as Question 11 have not been answered in this general situation.)