DISTRIBUTION OF MISSING CARDS – *A GOOD ESTIMATE*

1. Neatly complete the first eight rows of Pascal's Triangle:

ROW #	*PASCAL'S TRIANGLE*	Total
0	1	1
1	1 1	2
2	1 2 1	4
3	1 3 3 1	8
4	1 4 6 4 1	16
5	1 5 10 10 5 1	32
6	1 6 15 20 15 6 1	64
7	1 7 21 35 35 21 7 1	128

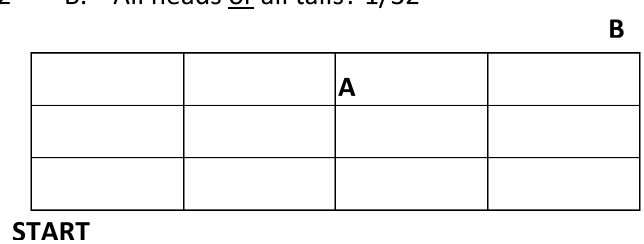
2. A family has 5 children. What is the probability that they have:

   A. Exactly three girls? 5/16 B. At least two boys? 13/16 C. 4G, 1B or 1G, 4B? 5/16

3. You flip one fair coin six times. What is the probability that you flip:

   A. 3 Heads and 3 Tails? 5/16 B. 4H, 2T or 2H, 4T? 15/32 B. All heads or all tails? 1/32

4. On this grid, you can only travel on the gridlines and only East (E) and North (N).



   A. How many different paths are there from START to A?

   4 blocks: 2 E's, 2 N's; 6 paths

   B. How many different paths are there from START to B? 7 blocks: 4 E's; 3 N's: 35 paths

   C. BONUS: What is the probability that a path from START to B passes through A? 6*3/35 = 18/35

Pascal's Triangle provides exact answers to Questions #2-4, above, but only *estimates* for the following questions.

5. Dummy and you have a total of 8 Spades. Estimate the probability that the other five Spades are divided:

   A. 2-3 or 3-2? 5/8 B. 1-4 or 4-1? 5/16 C. 0-5 or 5-0? 1/16

6. Dummy and you have a total of 9 Hearts. Estimate the probability that the other four Hearts are divided:

   A. 2-2? 3/8 B. 1-3 or 3-1? 1/2 C. 0-4 or 4-0? 1/8

7. Dummy and you have a total of 8 Diamonds, missing the Jack, 10, 9, 8, and 2. You lead the Ace, King, and Queen of Diamonds. Estimate the probability that you take all the Diamond tricks. _______

   The Diamonds must split 3-2 or 2-3: (10+10)/32 = **5/8**

8. Dummy and you have a total of 7 Clubs, missing the Jack, 10, 9, 8, 5, and 2. You lead the Ace, King, and Queen of Clubs. Estimate the probability that you take all the Club tricks. _______

   The Clubs must split 3-3: 20/64 = **5/16**

PASCAL'S TRIANGLE AS PERCENTS – STILL *A GOOD ESTIMATE*

It is often more convenient to express probabilities as percentages rather than as fractions. Of course, then each row must sum to 100%. Complete the first eight rows of Pascal's Percent Triangle. When needed, round to the nearest half of a percent. Due to rounding, some of your row sums will not be exactly 100%.

# of Cards	*PASCAL'S TRIANGLE as PERCENTS*	Total
0	**100**	**100**
1	**50 50**	**100**
2	**25 50 25**	**100**
3	**12.5 37.5 37.5 12.5**	**100**
4	**6 25 37.5 25 6**	**99.5**
5	**3 15.5 31 31 15.5 3**	**99**
6	**1.5 9 23 31 23 9 1.5**	**98**
7	**1 5 16 27 27 16 5 1**	**98**

1. If Dummy and you are missing six cards in a suit, estimate the percent probability that they split:

   A. 3-3? 31%   B. 4-2 or 2-4? 46%   C. neither A nor B? 21% [or 23%]

2. Dummy and you have 10 Spades, missing the Queen, 4, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Spade tricks. _______________

   They must split 1-2 or 2-1: 2*37.5 = **75%**

3. Dummy and you have 9 Diamonds, missing the Queen, Jack, 7, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Diamond tricks. _______________

   They must split 2-2: **37.5%**

4. Dummy and you have 8 Hearts, missing the Queen, Jack, 6, 3, and 2 . If you lead the Ace and King, estimate the percent probability that you take all the Heart tricks. _______________

   The only way is if the Q and J are a "doubleton" which can occur in two ways [E or W]: 2/32 = **6.125%**

5. Dummy and you have 9 Diamonds, missing the Queen, 7, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Diamond tricks. _____________

   They must split 2-2 OR the Queen must be a "singleton" which can occur in two ways: 6/16+2/16 = **50%**

<u>CARD DISTRIBUTIONS – THE EXACT PROBABILITIES</u>

For ***independent events*** such as boys/girls in a family [or flipping a fair coin], the probability of the next child [or coin] being "Girl" [or "tail"] remains 50% and is <u>not dependent</u> on the gender of the previous child [or result of previous coin flip]. Because of that, *Pascal's Triangle* provides their **<u>exact</u>** distributions and probabilities. However, the probability that the "next" card is a Heart <u>does change</u> based on whether the previous cards were or were not Hearts. Each probability **<u>IS dependent</u>** on previous cards. Therefore the distribution of cards is a *dependent event* and Pascal's Triangle only provides (good) estimates of the distributions of the cards.

<u>EXAMPLE</u>

<u>Dummy and you have 8 Hearts</u>. According to Pascal's Triangle, the probability that West has 3 Hearts and East has 2 Hearts is approximately 10/32 = **<u>31.25%.</u>** Now let's compute the <u>exact</u> probability.

West and East have a total of 26 cards of which **5** are Hearts and **21** are not Hearts. Let's calculate the probability that your West opponent has exactly 3 Hearts.

The <u>total number of different hands</u> West could have is C(26, 13) = 10,400,600 . The number of West hands with exactly 3 Hearts is C(5, 3)*C(21, 10) = 3,527,160 . Probability = C(5, 3)*C(21, 10)/C(26, 13) = **<u>33.9%</u>** [about 2.7% higher]

<u>EXACT Probabilities versus "Pascal Triangle Estimates"</u>

6A. If East and West have 5 Hearts, use the Table from page 2 to compute the estimated probability that they are split:

3-2 or 2-3: **<u>2*31% = 62%;</u>** 4-1 or 1-4: **<u>2*15.5% = 31%</u>** 5-0 or 0-5: **<u>2*3% = 6%</u>**

6B. Use the 'combination method' [above] to compute these exact probabilities.:

3-2 or 2-3: **<u>2*33.9% = 67.8%;</u>** {about 5% higher}

4-1 or 1-4: 2*C(5, 1)*C(21, 12)/C(26, 13) = **<u>28.3%</u>** {about 3% lower)

5-0 or 0-5: 2*C(5, 0)*C(21, 13)/C(26, 13) = **<u>3.9%</u>** {about 2% lower)