

DISTRIBUTION OF MISSING CARDS – A GOOD ESTIMATE

1. Neatly complete the first eight rows of Pascal's Triangle:

ROW #	PASCAL'S TRIANGLE	Total
0	1	1
1	1 1	2
2	1 2 1	4
3		
4		
5		
6		
7		

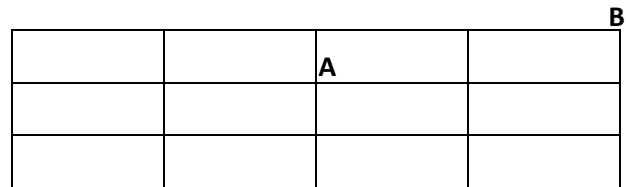
2. A family has 5 children. What is the probability that they have:

- A. Exactly three girls? _____ B. At least two boys? _____ C. 4G, 1B or 1G, 4B? _____

3. You flip one fair coin six times. What is the probability that you flip:

- A. 3 Heads and 3 Tails? _____ B. 4H, 2T or 2H, 4T? _____ B. All heads or all tails? _____

4. On this grid, you can only travel on the gridlines and only East (E) and North (N).



A. How many different paths are there from START to A?
[Hint: Two paths could be: NNEE or ENEN]

START

- B. How many different paths are there from START to B?
C. BONUS: What is the probability that a path from START to B passes through A?

Pascal's Triangle provides exact answers to Questions #2-4, above, but only estimates for the following questions.

5. Dummy and you have a total of 8 Spades. Estimate the probability that the other five Spades are divided:

- A. 2-3 or 3-2? _____ B. 1-4 or 4-1? _____ C. 0-5 or 5-0? _____

6. Dummy and you have a total of 9 Hearts. Estimate the probability that the other four Hearts are divided:

- A. 2-2? _____ B. 1-3 or 3-1? _____ C. 0-4 or 4-0? _____

7. Dummy and you have a total of 8 Diamonds, missing the Jack, 10, 9, 8, and 2. You lead the Ace, King, and Queen of Diamonds. Estimate the probability that you take all the Diamond tricks. _____

8. Dummy and you have a total of 7 Clubs, missing the Jack, 10, 9, 8, 5, and 2. You lead the Ace, King, and Queen of Clubs. Estimate the probability that you take all the Club tricks. _____

PASCAL'S TRIANGLE AS PERCENTS – STILL A GOOD ESTIMATE

It is often more convenient to express probabilities as percentages rather than as fractions. Of course, then each row must sum to 100%. Complete the first eight rows of Pascal's Percent Triangle. When needed, round to the nearest half of a percent. Due to rounding, some of your row sums will not be exactly 100%.

# of Cards	<i>PASCAL'S TRIANGLE as PERCENTS</i>	Total
0	100	100
1	50 50	100
2	25 50 25	
3		
4		
5		
6		
7		

- If Dummy and you are missing five cards in a suit, estimate the percent probability that they split:
 A. 3-2 or 2-3? _____ B. 4-1 or 1-4? _____ C. 4-0 or 0-4? _____
- Dummy and you have 10 Spades, missing the Queen, 4, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Spade tricks. _____
- Dummy and you have 9 Diamonds, missing the Queen, Jack, 7, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Diamond tricks. _____
- Dummy and you have 8 Hearts, missing the Queen, Jack, 6, 3, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Heart tricks. _____
- Dummy and you have 9 Diamonds, missing the Queen, 8, 7, and 2. If you lead the Ace and King, estimate the percent probability that you take all the Diamond tricks. _____

Note: The answers to #3 and #5 are NOT the same!

CARD DISTRIBUTIONS – THE EXACT PROBABILITIES

For **independent events** such as boys/girls in a family [or flipping a fair coin], the probability of the next child [or coin] being “Girl” [or “tail”] remains 50% and is not dependent on the gender of the previous child [or result of previous coin flip]. Because of that, *Pascal’s Triangle* provides their exact distributions and probabilities. However, the probability that the “next” card is a Heart does change based on whether the previous cards were or were not Hearts. Each probability **IS dependent** on previous cards. Therefore the distribution of cards is a *dependent event* and Pascal’s Triangle only provides (good) estimates of the distributions of the cards.

EXAMPLE

Dummy and you have 8 Hearts. According to Pascal’s Triangle (above), the probability that West has 3 Hearts and East has 2 Hearts is approximately $10/32 = 31.25\%$. Now let’s compute the exact probability.

West and East have a total of 26 cards of which ____ are Hearts and ____ are not Hearts. Let’s calculate the probability that your West opponent has exactly 3 Hearts.

The total number of different hands West could have is $C(26, 13) = 10,400,600$. The number of West hands with exactly 3 Hearts is $C(5, 3) * C(21, 10) = 3,527,160$. Probability = $C(5, 3) * C(21, 10) / C(26, 13) = 33.9\%$ [about 2.7% higher]

EXACT Probabilities versus “Pascal Triangle Estimates”

6A. If East and West have 5 Hearts, use the Table from page 2 to compute the estimated probability that they are split:

3-2 or 2-3: **2*31% = 62%**; 4-1 or 1-4: _____; 5-0 or 0-5: _____

6B. Use the ‘combination method’ [above] to compute these exact probabilities.:

3-2 or 2-3: **2*33.9% = 67.8%**; 4-1 or 1-4: _____; 5-0 or 0-5: _____

NOTE: Most bridge players never calculate these approximate or exact probabilities. From teachers or books, they learn and memorize the probabilities of the most common distributions. For example, it is often sufficient to know that with 4 missing cards: **a 3-1 or 1-3 split is more likely than a 2-2 split.** **OR** With 5 missing cards: **a 3-2 or 2-3 split is much more than all other splits combined.**

An excellent reference is at https://www.lajollabridge.com/LJUnit/Education/Art_of_Being_Lucky.pdf