Burnside's Lemma October 14, 2018

1. (a) What if we had 6 chairs instead of 10? How many ways are there to arrange 6 chairs around a table with two color choices, if we consider setups to be the same if we can rotate one to get the other? What if we had 8 chairs?

   (b) What if we had n color choices instead of 2? Try it for 6 chairs and for 8 chairs.

   (c) Can you show that your answers to part 1b are whole numbers another way?

2. Try the previous problem again, but this time consider setups to be the same if we can rotate *or reflect* one to get the other.

3. For 6 chairs and two colors, find a pair of setups that counted as different in problem 1 but the same in problem 2. Can you find any other pairs of setups like that?

4. What if we were painting a tetrahedron instead of a cube?

   (a) List the symmetries of a tetrahedron. What kinds of symmetries are there, and how many are there of each kind?

   (b) How many ways are there to paint a tetrahedron with three colors, up to rotation?

   (c) How many ways are there to paint a tetrahedron with n colors, up to rotation?

   (d) Check your work to part 4c by plugging in $n = 2$ and then explicitly listing all of the ways to paint the tetrahedron with two colors.

   (e) Can you show that your answer to part 4c is a whole number another way?