MO-ARML --- January, 2019 -- AMC 10 Practice

The AMC 10A and AMC 10B are 25-question tests offered each February by the Mathematical Association of America.The time limit is 75 minutes and calculators are NOT allowed. Each question is multiple choice with 5 options. *The scoring is: Correct answer: 6 points; No answer: 1.5 points; Incorrect answer: 0 points.* With this scoring, students should NOT 'just guess' any answers.

More information can be found at: https://www.maa.org/math-competitions

Past tests and solutions are at: https://artofproblemsolving.com/wiki/index.php/AMC_Problems_and_Solutions

These 15 questions are selected from #1 – 17 on the 2007 AMC 12B and 2011 10B tests.

1. Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?

    A. 678   B. 768   C. 786   D. 87   E. 876

2. A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?

    A. 22   B. 24   C. 25   D. 26   E. 28

3. The point O is the center of the circle circumscribed about ΔABC with $\angle AOB = 140^o$ and $\angle BOC = 120^o$.

    What is the degree measure of $\angle ABC$?

    A. 35   B. 40   C. 45   D. 50   E. 55

4. LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. At the end of the trip, for the joint expenses LeRoy had paid \$A and Bernardo had paid \$B with $A < B$. How many dollars must LeRoy give Bernardo so that they share their joint expenses equally?

    A. $\frac{A + B}{2}$ B. $\frac{A - B}{2}$ C. $\frac{B - A}{2}$ D. B – A   E.   A + B

5. The AMC contests are scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. Of the 25 problems, Sarah has decided to leave the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

    A. 13   B. 14   C. 15   D. 16   E. 17

6. The sum of two angles of a triangle is 6/5 of a right angle and one of these angles is 30⁰ larger than the other. What is the degree measure of the largest angle of the triangle?

    A.   69   B. 72   C. 90   D. 102   E. 108

7. If it is sunny and at least 80⁰ F at a certain beach, that beach will be crowded. Since the beach was not crowded on June 10, what can be concluded about the weather conditions on June 10?

    A.   The temperature was cooler than 80⁰ F and it was not sunny.

    B.   The temperature was cooler than 80⁰ F or it was not sunny.

    C.   If the temperature was at least 80⁰ F, then it was sunny.

    D.   If the temperature was cooler than 80⁰ F, then it was sunny.

    E.   If the temperature was cooler than 80⁰ F, then it was not sunny.

8. Tom's age is T years which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N?

   A. 2   B. 3   C. 4   D. 5   E. 6

9. A function *f* has the property that $f(3x - 1) = x^2 + x + 1$ for all real numbers *x*. What is $f(-7)$?

   A. 54/9   B. – 79/9   C. – 5   D. 3   E. 43

10. Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave, two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?

    A. 4   B. 6   C. 8   D. 190   E. 12

11. $AB = 5$; $BC = 3$; $CA = 4$. E is on BC and D is on AB such that ED is perpendicular to AB. If the area of $\triangle EBD$ is one-third of the area of $\triangle ABC$, what is BD?

    A. 4/3   B. $\sqrt{5}$ C. 9/4   D. $\frac{4\sqrt{3}}{3}$ E. 5/2

12. There are 52 people in a room. What is largest value of *N* such that the statement "*At least N people have birthdays in the same* month," is always true?

    A. 2   B. 3   C. 4   D. 5   E. 12

13. Two real numbers are selected independently at random from the interval [– 20, 10]. What is the probability that the product of those two numbers is greater than zero?

    A. 1/9   B. 1/3   C. 4/9   D. 5/9   E. 2/3

14. The infinite geometric series $a + ar + ar^2 + \ldots$ has a sum of 7 and the terms involving odd powers of *r* have a sum of 3. What is $a + r$?

    A. 4/3   B. 12/7   C. 3/2   D. 7/3   E. 5/2

15. Each face of a regular tetrahedron is painted either red, white, or blue. Two colorings are considered indistinguishable if two congruent tetrahedra with those colorings can be rotated so that their appearances are identical. How may distinguishable colorings are possible?

    A. 15   B. 18   C. 27   D. 54   E. 81

16. Pentagon ABCDE is inscribed in a circle such that BE is a diameter, CD is parallel to BE, and AB is parallel to DE. If the angles AEB and ABE are in the ratio 4:5, what is the degree measure of angle BCD?

    A. 120   B. 125   C. 130   D. 135   E. 140

1. E   876 square feet

2. B   24 mpg   Compute "*total distance divided by total gallons used*"

3. D   50   The Central Angle Theorem can be useful in this problem.

4. C   (B – A)/2   Hmmm, if B paid $260 and A paid $200, how much of the total "should" A have paid?

5. D   16

6. B   72   I set up two equations in terms of angle measures A and B.

7. B   Use 'common sense' or use that the negation of *"P implies Q"* is *"Not Q implies not P."*
                      Note that "*the beach being not crowded*" means that it is not <u>both</u> hot and sunny.

8. D   5   Set up two equations for T and N. You cannot solve for T or N but you can solve for T/N.

9. D   3   For what x does $3x – 1 = – 7$?

10. C   8   Set up two equations in B and G. Or consider: S*ince the total number of students in the group remain unchanged, the 2 girls who left represented what per cent of the original group?*

11. D $\frac{4\sqrt{3}}{3}$ Use similar triangles to compute DE/BD; then area formulas.

12. D   5   A nice intro to the Pigeon Hole principle. Can you place 3 people in each month? 4? 5?

13. D   5/9   Consider the TWO cases.

14. E   5/2   Use $S = a/(1 – r)$ twice. For a shorter solution, factor r from: $3 = ar + ar^3 + ar^5 +...$ to create the series of even terms!

15. A   15   Four cases for (r, w, b): (4,0,0), (3,1,0), (2,2,0), (2,1,1). How often can each occur?

16. C   130   You can use the angle properties of *cyclic quadrilaterals, isosceles trapezoid,* or *central angles.*