# MO-ARML

*Modular Mathematics*
*Worksheet Problems*

Name ________________________________

1. Noting that 2457 can be found to be divisible by 9 by a standard shortcut, discover a property of the mod 9 relation using the numbers 392, 775, and 9201. Prove it.

2. Find: **a]** 14648 mod 11 **b]** 64 mod $(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)$.

3. If $x \equiv 500 \pmod{17}$, find the smallest prime solution greater than 2000.

4. When $n$ is divided by 5, the remainder is 1. What is the remainder when $7n$ is divided by 5?

5. Solve for $x$ if $1233x + 45 \equiv 9090 \pmod{24}$.

6. Find $x$ if $x \equiv 6^{1000} \pmod{23}$.

7. Use the Euclidean algorithm to find GCD(1529, 14039).

8. Find $x$ if $3x \equiv 4 \pmod{7}$.

9. Find $x$ if $4x \equiv 5 \pmod{8}$.

10. What is the square root of 5 mod 61?

11. Find all integral solutions to $x^2 + y^2 = 1000003$.

12. Prove: $(m + 1)^2 \equiv (m - 1)^2 \pmod{2m}$, $m \in \mathbf{Z}$.

13. Find the smallest value for $x$ if $7^x \equiv 1 \pmod{18}$.

14. What is the size of the largest subset $S$ of $\{1, 2, 3, \ldots, 50\}$, such that no pair of distinct elements of $S$ has a sum divisible by 7?

15. Prove: if $\underline{abc} \equiv 0 \pmod{19}$, then $\underline{cba}_2 \equiv 0 \pmod{19}$.

16. Prove: $(1 + 4x)^2 \equiv 1 + 8x^2 \pmod{2}$, $x \in \mathbf{Z}$.

17. Find the smallest $x$ in $\mathbf{Z}^+$ such that it has a remainder of 9 when divided by 10, a remainder of 8 when divided by 9, a remainder of 7 when divided by 8, …, and a remainder of 1 when divided by 2.

18. If $x \equiv 5 \pmod{7}$, then what is $1/x$ congruent to mod 7?

19. What is the other factor $k$ which satisfies $k\,(x - 2) \equiv 5x^3 + x^2 + 5x + 2 \pmod{7}$?

20. Find the smallest $x$ in $\mathbf{Z}^+$ such that $x \equiv 3 \pmod{11}$ and $x \equiv 5 \pmod{6}$.

21. Solve for $x$ if $14x \equiv 10 \pmod{12}$.

22. Solve for $x$ if $x \equiv 4 \pmod{5}$, $x \equiv 7 \pmod{8}$, and $x \equiv 3 \pmod{9}$.

23. Compute the date for Easter in 2020.