MO-ARML

Modular Worksheet Hints, Answers or Solutions

Name ______________________________

1. [each sum pretty much sums it up]

2. **a]** $\boxed{7}$ [11^4 is ?] **b]** [patterns in base 2]

3. $\boxed{2081}$ [adding on 510 3 times gets you to 2030, but be careful about checking factors]

4. $\boxed{2}$ [if $n \equiv 1 \pmod 5$, then $7n = 7 \cdot 1 = 7 \equiv 2 \pmod 5$]

5. $1233x \equiv 9045 \pmod{24} \rightarrow (1224 + 9)x \equiv (9024 + 21) \pmod{24} \rightarrow 9x \equiv 21 \pmod{24} \rightarrow$
   $3x \equiv 7 \equiv 15 \pmod 8$ [risky division] $\rightarrow x \equiv 5 \pmod 8 \rightarrow \boxed{x \in \{\ldots, 5, 13, \ldots\}}$ √

6. $6^{23} \equiv 6 \pmod{23} \rightarrow 6^{22} \equiv 1 \pmod{23} \rightarrow$ since $6^{990} \equiv (6^{22})^{45} \equiv 1 \pmod{23} \rightarrow$
   $6^{1000} = 6^{990} \cdot 6^{10} \equiv 6^{10} = 7776^2 \equiv 2^2 = 4 \pmod{23} \rightarrow \boxed{x \in \{\ldots, 4, 27, \ldots\}}$

7. $14039 = 9 \cdot 1529 + 278 \rightarrow 1529 = 5 \cdot 278 + 139 \rightarrow 278 = 2 \cdot 139 + 0 \rightarrow (14039, 1529) = \boxed{139}$

8. $\boxed{6 + 7k,\ k \in \mathbf{Z}}$

9. $\boxed{\emptyset}$

10. $5 \equiv (5 + 610 + 61) \equiv 676 \equiv 26^2 \pmod{61}$; square root is $\boxed{26}$ or $\boxed{26 + 61k \text{ or } 35 + 61k,\ k \in \mathbf{Z}}$

11. Since $1000003 \equiv 3 \pmod 4$, $\boxed{\emptyset}$

12. [expand]

13. by trial-and-error, 7^0, 7^1, 7^2, 7^3, 7^4 and 7^5 don't work, but 7^6 does; so $\boxed{6}$

14. partition S into subsets: $x \equiv 0 \pmod 7$, $x \equiv 1 \pmod 7$, $x \equiv 2 \pmod 7$, …, $x \equiv 6 \pmod 7$, including all remainders under mod 7; considering $x \equiv 3 \pmod 7$ and $x \equiv 4 \pmod 7$, their sums are always 7, so discard one; for each of 1 to 50, from $x \equiv 0 \pmod 7$, $x \equiv 1 \pmod 7$, $x \equiv 2 \pmod 7$, $x \equiv 3 \pmod 7$, we can choose $1 + 8 + 7 + 7 = \boxed{23}$

15. using mod 19: $a \cdot 100 + b \cdot 10 + c \equiv 4(a \cdot 100 + b \cdot 10 + c) \equiv a \cdot 20 \cdot 20 + b \cdot 2 \cdot 20 + c \cdot 4 \equiv 4c + 2b + a \equiv$ $\underline{cba}_2 \pmod{19}$ QED

16. [expand and consider each term]

17. $\boxed{2519}$ [if starting with factorial, watch for unneeded factors]

18. since want inverse of x, need $x \cdot 1/x = 1 \equiv 15 = 3 \cdot 5 \pmod 7$, so $\boxed{1/x = 3}$

19. $\boxed{5x^2 + 4x + 6}$ [use polynomial division under mod 7]

20. [using CRT, valid excluded multiples yield $36 + 11 = 47$] $\boxed{47}$

21. $14x \equiv 2x \equiv 10 \pmod{12} \rightarrow x \equiv 5 \pmod 6$ [risky division] $\rightarrow \boxed{5 + 6k,\ k \in \mathbf{Z}}$ √

22. [using CRT again, valid excluded multiples yield $144 + 135 + 120 = 399$] $\boxed{39 + 360k,\ k \in \mathbf{Z}}$

23. $\boxed{\text{Sunday, April 12, 2020}}$