Target Round 11901

Name _____

Grade _____

School _____

Place ID Sticker Inside This Box

Questions 1 & 2

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

1. What is the smallest positive integer such that its base 2 representation has at least two more digits than its base 3 representation? Express your answer in base 10.

1.

2. The 24 members of student government are divided into three parties: 12 people are in the Free Pizza party, 9 are in the Free Chips party, and 3 are in the No Free Food party. To resolve their differences, they decide that each member must have a meeting with everyone who disagrees with them, and exactly two people who agree with them; each meeting is between only two people. Compute the number of resulting meetings.

2.

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Questions 3 & 4

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

3. Twelve people are playing a 32-race match in a racing game, in which points are awarded to each player based on their finishing position in each race (with players who finish faster awarded more points). The two best players by far are Jaeyoung and Huy; for the first N races, Jaeyoung gets first and Huy gets second. It takes N races for Jaeyoung to be guaranteed to beat Huy on overall points, so after that, he starts goofing around, getting last in the rest of the races while Huy gets first. If first place is worth 15 points, second is worth 12 points, and last is worth no points, find N.

3.

4. The point $(3, 4, 12)$ is rotated about the origin to a point $(x, y, 0)$. This point is shifted 1 unit in the positive x-direction, and then rotated about the origin to the point $(0, 0, 4\sqrt{10})$. Find $|y|$.

4.

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Questions 5 & 6

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

5. Huy gets around college either by scooter or by walking. He scooters 4 times as fast as he walks. On his 500 foot trip to get from one class to another, he starts on his scooter but then reaches a walk-only zone and has to walk the rest of the way. The next week on the same trip, he walks the entire way, and it takes him twice as long as it did previously. How far, in inches, did he travel in the walk-only zone? (1 foot = 12 inches)

5.

6. Pranav and Alan are both rolling standard 6-sided dice. Pranav rolls two dice and multiplies the two values together, while Alan rolls one die and squares the value. Let P and A be the expected values of Pranav's and Alan's results, respectively. Compute $P - A$. Express your answer as a common fraction.

6.

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Questions 7 & 8

Score #1 _____ Scorer's Initials _____

Score #2 _____ Scorer's Initials _____

7. Find the median of the imaginary parts of the non-real roots of $x^{2018} + 2017x^2 + 1 = 0$.

7.

8. For prime positive integers p, define p? to be the product of all primes less than or equal to p. The equation p? + 1 = $2^p - 1$ holds for small values of p, but not in general. Find the largest integer N such that N divides (p? + 1) − ($2^p - 1$) for all p.

8.

Answer Key 11901

Sprint Round Answers

1. C
2. B
3. D
4. C
5. B
6. C
7. D
8. A
9. D
10. D
11. C
12. D
13. C
14. D
15. D
16. B
17. A
18. D
19. E
20. E
21. D
22. E
23. D
24. A
25. E
26. A
27. D
28. E
29. E
30. A

Target Round Answers

1. 8
2. 195
3. 27
4. 12
5. 2000
6. $-\frac{35}{12}$
7. 0
8. 12

Team Round Answers

1. 20
2. 60
3. 265
4. $\frac{525}{22}$
5. 9801
6. 34
7. 466
8. 162
9. 5184
10. $3\sqrt{3}$

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Solutions 11901

Target Round Solutions

1. Clearly, $100_2 = 11_3$ is too small, so we try $1000_2 = 22_3$, and that works and we are done. The integer is 8.

2. Lets divide this into cases. First consider the number of meetings between two members of the same party. Each of the 12 people in the Free Pizza party meet with 2 other people, so there are 24 meetings. However, this counts each meeting twice, once from the perspective of each member, so our actual count is just 12. Similarly, there are 9 meetings between members of the Free Chips party, and 3 meetings between members of the No Free Food party. Between the Free Pizza party and Free Chips party, each Free Pizza member has 9 meetings, so there are 108 meetings in total. (Why are we not double counting?) Similarly, there are 36 meetings between the Free Pizza party and No Free Food party, and 27 meetings between the Free Chips and No Free Food party. Adding all these numbers up, we have 195 meetings in total.

3. Converting to numbers gives $15N > 12N + 15(32 - N)$. This is equivalent to $18N > 15 \cdot 32$, or $N > \frac{80}{3}$. Thus, the smallest value of N for which Jaeyoung is guaranteed to win is $N = 27$.

4. Rotating about the origin does not change distance from the origin, so we get $x^2 + y^2 = 3^2 + 4^2 + 12^2 = 169$ and $(x+1)^2 + y^2 = (4\sqrt{10})^2 = 160$. Subtracting gives $2x + 1 = -9$, or $x = -5$. Thus $y = \pm\sqrt{169 - (-5)^2} = \pm\sqrt{144} = \pm 12$.

5. Suppose the walk-only zone has length d, and Huy walks at a rate of r. Then the total time of the first trip is $\frac{d}{r} + \frac{500-d}{4r}$, and the total time of the second trip is $\frac{500}{r}$, so $\frac{500}{r} = 2(\frac{d}{r} + \frac{500-d}{4r})$. Rearranging gives $\frac{500-2d}{r} = \frac{500-d}{2r}$; multiplying by 2r gives $1000 - 4d = 500 - d$, or $d = \frac{500}{3}$ feet, or 2000 inches.

6. The expected roll on a single die is $\frac{7}{2}$, so Pranav's expected value is the square of that value, or $\frac{49}{4}$. Alan's expected value is $\frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2) = \frac{1}{6} \cdot \frac{6 \cdot 7 \cdot 13}{6} = \frac{91}{6}$. Thus, $P - A = -\frac{91}{6} + \frac{49}{4} = (12 - 15) + (\frac{1}{4} - \frac{1}{6}) = -3 + \frac{1}{12} = -\frac{35}{12}$.

7. It is well-known that for polynomials with real coefficients, if $a + bi$ is a root of the polynomial and $b \neq 0$, then $a - bi$ is also a root of the polynomial. Thus if we organize the imaginary parts in order, the median would have to be 0.

8. Checking small values, we find for p = 7, p? + 1 = 211 while $2^p - 1 = 127$. Thus N must divide $211 - 127 = 84 = 2^2 * 3 * 7$. Since p? is always 2 times a product of odd numbers, p? $\equiv$ 2 mod 4, so (p? + 1) $-$ ($2^p - 1$) = p? $- 2^p + 2 \equiv$ p? + 2 $\equiv$ 0 mod 4 for p $\geq$ 3. Thus 4 divides our difference, and since 8 $\nmid$ 84, 8 cannot divide N, while 4 must. When p is odd, $2^p \equiv 2$ mod 3 by Fermat's Little Theorem. Thus (since p $\geq$ 3, and thus 3 $\mid$ p?) (p? + 1) $-$ ($2^p - 1$) $\equiv 2 - 2^p \equiv$ 0 mod 3, so N is divisible by 3. Since 9 $\nmid$ 84, we know 9 does not divide N. To check if 7 divides N, we plug in p = 11 to get (p? + 1) $-$ ($2^p - 1$) = 2311 $-$ 2047 = 264 = $2^3 * 3 * 11$, so 7 cannot divide N. Since N divides 84, no other prime can divide N, and therefore N = $2^2 * 3$ = 12. Note that p? is the aptly named primorial function.

